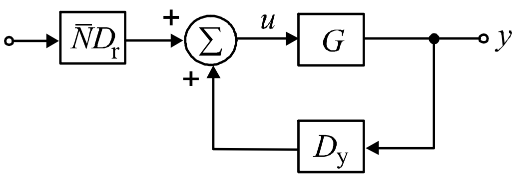


General closed-loop system block diagram for Problem 7.55.

Problems and Solutions for Section 7.10: Integral Control and Robust Tracking

56. Assume that the linearized and time-scaled equation of motion for the ball-bearing levitation device is $\ddot{x} - x = u + w$. Here w is a constant bias due to the power amplifier. Introduce integral error control, and select three control gains $\mathbf{K} = [\ K_1 \quad K_2 \quad K_3\ ]$ so that the closed-loop poles are at -1 and $-1 \pm j$ and the steady-state error to w and to a (step) position command will be zero. Let $y = x$ and the reference input $r \triangleq y_{\text{ref}}$ be a constant. Draw a block diagram of your design showing the locations of the feedback gains K_i. Assume that both $\dot{x}$ and x can be measured. Plot the response of the closed-loop system to a step command input and the response to a step change in the bias input. Verify that the system is type 1. Use MATLAB (Simulink) software to simulate the system responses.

Solution:

The equations of motion are given by,

$$\begin{aligned} \ddot{x} - x &= u + w, \\ \dot{w} &= 0. \end{aligned}$$

A realization of these equations is,

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w, \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}. \end{aligned}$$

In order to incorporate integral control, we augment the state vector with an integral state, x_I, such that,

$$\dot{x}_I = y - r.$$

With the augmented state vector, $\mathbf{z} = [x_I \ x \ \dot{x}]^T$, the augmented state matrices become,

$$\mathbf{F}_a = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{G}_a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{H}_a = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

The design of the state feedback vector, $\mathbf{K}$, is now done using the above augmented state matrices. For closed-loop poles of $s = -1, -1 \pm j$,

$$\det(s\mathbf{I} - \mathbf{F}_a + \mathbf{G}_a\mathbf{K}) = 0,$$

when,

$$\mathbf{K} = [\ K_1 \quad K_2 \quad K_3\ ] = [\ 2 \quad 5 \quad 3\ ].$$

This result can be verified using the MATLAB `place` command.

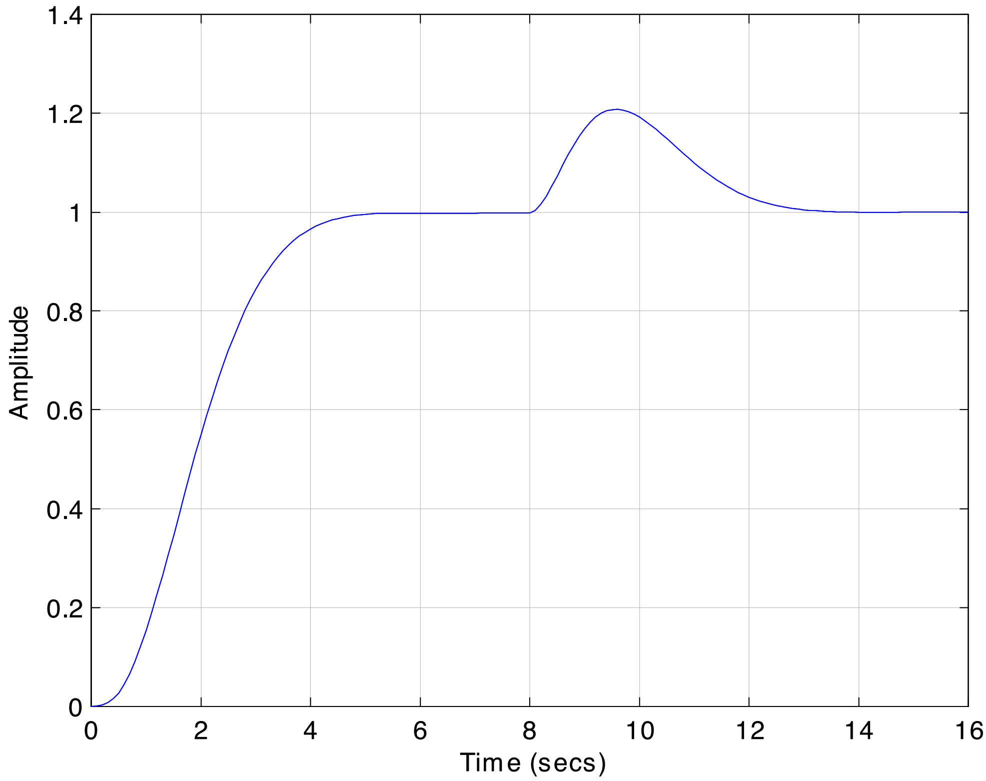
The closed-loop system is given by,

$$\begin{aligned}\dot{\mathbf{z}} &= (\mathbf{F}_a - \mathbf{G}_a\mathbf{K})\mathbf{z} + \mathbf{G}_a w + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} r, \\ y &= \mathbf{H}_a\mathbf{z}.\end{aligned}$$

To show that the system is Type I, show that $y = 0$ for any constant w in the steady-state, i.e., $\dot{\mathbf{z}} = 0$. For the closed-loop system we have,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_1 & 1 - K_2 & -K_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w.$$

This immediately gives $z_2 = 0$ and $y = z_2 = 0$. Thus, in steady-state $y = 0$ for any constant w in the steady-state. The figure below shows a simulation of the closed-loop system to a commanded step r, at $t = 0$. At $t = 8$, a step in the constant bias w is applied. This figure was generated using the MATLAB `lsim` command.



Problem 7.56: Response of closed-loop system to a unit step input at $t = 0$ and step disturbance at $t = 8$.

The simulation of the closed-loop system in Simulink is shown on the next page.