Regelungstechnik Übungen

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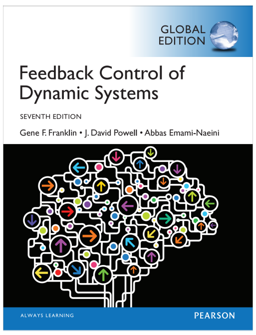
Inhaltsverzeichnis

Literatur

[FPE15] Gene F. Franklin, J. David Powel und Abbas Emami-Naeini. *Feedback Control of Dynamic Systems*. 7th global edition. Pearson Prentice Hall, 2015.



1 Einführung Übungen

Aufgabe 1.1: Review-Fragen

[FPE15, Seite 39f]

1. Was sind die wesentlichen Komponenten eines Regelungssystems?
2. Was ist der Zweck eines Sensors?
3. Was ist der Zweck eines Stellglieds?
4. Was ist der Zweck eines Reglers? Geben Sie die Ein- und Ausgänge des Reglers an.
5. Warum haben die meisten Sensoren einen elektrischen Ausgang, unabhängig von der physikalischen Natur der gemessenen Variable?
6. Was ist der Unterschied zwischen einer Steuerung und einer Regelung?

2 Das dynamische Verhalten von Systemen Übungen

Aufgabe 2.1: Review-Fragen

[FPE15, Seite 169]

1. Was ist die Definition von *Übertragungsfunktion (transfer function)*?
2. Welche Eigenschaften haben Systeme, die sich mit Transferfunktionen beschreiben lassen?
3. Was besagt der 2. Grenzwertsatz (Final Value Theorem) und wozu wird er in der Regelungstechnik angewendet?

Aufgabe 2.2: Laplace-Transformation

Lösen Sie alle Aufgaben 3.2 bis 3.11 in [FPE15].

Errata

3.2 (c) $f(t) = (3t + 4)^3 + 5t$

3.2 (d) $f(t) = (t + 1)^2$

3.2 (e) $f(t) = \sinh t$

3.4 (e) $f(t) = 2\delta(t) + 3t \sin 4t$

3.8(a) $F(s) = \dfrac{1}{s(s+2)^2}$

3.8(b) $F(s) = \dfrac{2s^2 + s + 1}{s^3 - 1}$

3.8(c) $F(s) = \dfrac{2(s^2 + s + 1)}{s(s+1)^2}$

3.8(d) $F(s) = \dfrac{s^3 + 2s + 4}{s^4 - 16}$

3.8(e) $F(s) = \dfrac{2(s+2)(s+5)^2}{(s+1)(s^2+4)^2}$

3.8(f) $F(s) = \dfrac{(s^2 - 1)}{(s+1)^2}$

REVIEW QUESTIONS

3.1 What is the definition of “frequency response”?

3.2 What are the three domains used to study the dynamic response of a system?

3.3 If $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$, then what will be the result of $\int_{-\infty}^{\infty} f(\tau)\, \delta(t-\tau)\, d\tau$?

3.4 State the Initial Value Theorem.

3.5 What is the most common use of the FVT in control?

3.6 Consider a generic system to have an input of $R(s)$. Let the output of this system be $Y(s)$ and let the system be defined by the function $H(s)$. What is the transfer function of this system? (Write the answer in terms of input–output relation.)

3.7 What is the Laplace transform of the impulse function?

3.8 What is the Laplace transform of the sinusoid function?

3.9 What is Mason’s gain formula?

3.10 What is meant by the natural frequency of a control system?

3.11 What is the main drawback of the Routh–Hurwitz criterion?

3.12 What is the major disadvantage of root-locus plots?

PROBLEMS

Problems for Section 3.1: Review of Laplace Transforms

3.1 Show that, in a partial-fraction expansion, complex conjugate poles have coefficients that are also complex conjugates. (The result of this relationship is that whenever complex conjugate pairs of poles are present, only one of the coefficients needs to be computed.)

3.2 Find the Laplace transform of the following time functions:

(a) $f(t) = 1 + 5t$

(b) $f(t) = 1 + 2t + 3t^2 + 4t^3$

(c) $f(t) = (3t + 4)^3 + 5^t$

(d) $f(t) = 3\sqrt{t} + 4/\sqrt{t}$

(e) $f(t) = t^{-5/2} + t^{5/2}$

3.3 Find the Laplace transform of the following time functions:

(a) $f(t) = 4 \sin 3t$

(b) $f(t) = \cos 2t + 4 \sin 5t + e^{-2t} \cos 7t$

(c) $f(t) = t + e^{-3t} \cos 5t$

3.4 Find the Laplace transform of the following time functions:

(a) $f(t) = t \cos t$

(b) $f(t) = t \sin 3t$

(c) $f(t) = te^{-2t} + 3t \sin 2t$

(d) $f(t) = t \cos 2t + 3t \sin t$

(e) $f(t) = 2\delta(t) + 3t \sin 4t$

3.5 Find the Laplace transform of the following time functions (* denotes convolution):

(a) $f(t) = \cos 2t \cos 5t$

(b) $f(t) = 3\cos^2 t + 2\sin^2 t$

(c) $f(t) = t^2 \cos t$

(d) $f(t) = \cos t^* \sin t$

(e) $f(t) = \int_0^t \sin(t-\tau)\sin\tau \, d\tau$

3.6 Given that the Laplace transform of $f(t)$ is $F(s)$, find the Laplace transform of the following:

(a) $g(t) = f(t) \sin t$

(b) $f(t) = \int_0^{t_1}\int_0^{t_2}\int_0^{t_3} f(t_3)\, dt_3\, dt_2\, dt_1$

3.7 Find the time function corresponding to each of the following Laplace transforms using partial-fraction expansions:

(a) $F(s) = \frac{1}{s(s+2)}$

(b) $F(s) = \frac{3}{s(s+1)(s+2)}$

(c) $F(s) = \frac{4s+5}{(s+1)^2(s+2)}$

(d) $F(s) = \frac{s+2}{s^2(s+3)}$

(e) $F(s) = \frac{1}{s(s+1)(s+2)(s+3)}$

(f) $F(s) = \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$

(g) $F(s) = \frac{2s^2+5s-4}{s^3+s^2-2s}$

(h) $F(s) = \frac{3s+2}{s^2-s-2}$

(i) $F(s) = \frac{s^2}{(s^2+1)(s^2+4)}$

(j) $F(s) = \frac{(3s+1)e^{-3s}}{(s-1)(s^2+1)}$

3.8 Find the time function corresponding to each of the following Laplace transforms:

(a) $F(s) = \log(s + a/s + b)$

(b) $F(s) = \log[1 - (a/s)^2]$

(c) $F(s) = \cot^{-1}(s/a)$

(d) $F(s) = \log\sqrt{s^2 + \frac{1}{s^2} + 4}$

(e) $F(s) = \log\left[\frac{s^2+4}{s(s+4)(s-4)}\right]$

(f) $F(s) = \tan^{-1}\left(\frac{2}{s^2}\right)$

(g) $F(s) = \cot^{-1}\left(\frac{s+a}{b}\right)$

3.9 Solve the following ODEs using Laplace transforms:

(a) $\dddot{y}(\dot{t}) + 2\ddot{y}(t) - \dot{y}(t) - 2y(t) = 0$ given $y(0) = \dot{y}(0) = 0$ and $\ddot{y}(0) = 6$

(b) $\ddot{y}(t) + 4\dot{y}(t) + 4y(t) = e^{-t}$ given $y(0) = \dot{y}(0) = 0$

(c) $\ddot{y}(t) - 2\dot{y}(t) + y(t) = e^{2t}$ given $y(0) = \dot{y}(0) = -1$

(d) $\ddot{y}(t) + 2\dot{y}(t) + y(t) = 3te^{-t}$ given $y(0) = 4, \dot{y}(0) = 2$

(e) $\ddot{y}(t) + 2\dot{y}(t) + 2y(t) = 5\sin\ t$ given $y(0) = \dot{y}(0) = 0$

(f) $\ddot{y}(t) + 6\dot{y}(t) + 9y(t) = 12t^2e^{-3t}$ given $y(0) = \dot{y}(0) = 0$

3.10 Using the convolution integral, find the step response of the system whose impulse response is given below and shown in Fig. 3.44:

$$h(t) = \begin{cases} 2te^{-2t}, & t \geq 0, \\ 0, & t < 0. \end{cases}$$

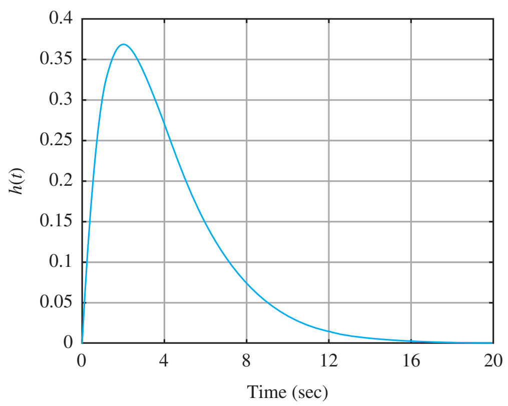


Figure 3.44
Impulse response for Problem 3.10

3.11 Using the convolution integral, find the step response of the system whose impulse response is given below and shown in Fig. 3.45:

$$h(t) = \begin{cases} 2, & 0 \leq t \leq 2, \\ 0, & t < 0 \text{ and } t > 2. \end{cases}$$

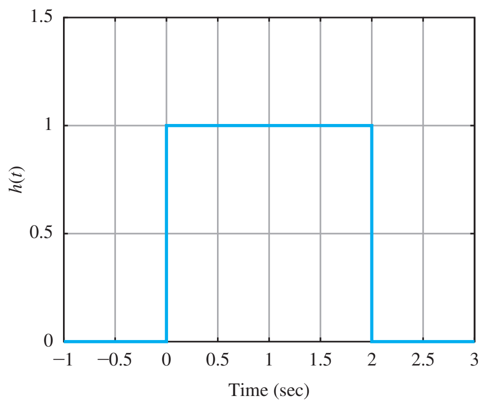


Figure 3.45
Impulse response for Problem 3.11

3.12 Consider the standard second-order system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}.$$

3 Polstellenlage

Aufgabe 3.1: Review-Fragen

[FPE15, Seite 169]

1. In welchen drei Domänen lässt sich das Zeitverhalten eines Systems untersuchen?
2. Was ist der (graphische) Zusammenhang zwischen der gedämpften / ungedämpften Eigenkreisfrequenz ω_d / ω_n und den Polen eines Systems zweiter Ordnung?
3. Mit welcher Frequenz schwingt ein gedämpftes, schwingungsfähiges System zweiter Ordnung, das mit einem Sprung oder einem Impuls angeregt wird, wenn ansonsten keine äußeren Kräfte wirken?

Aufgabe 3.2: Reglerauslegungen

Lösen Sie alle Aufgaben 3.26 bis 3.32 aus [FPE15].

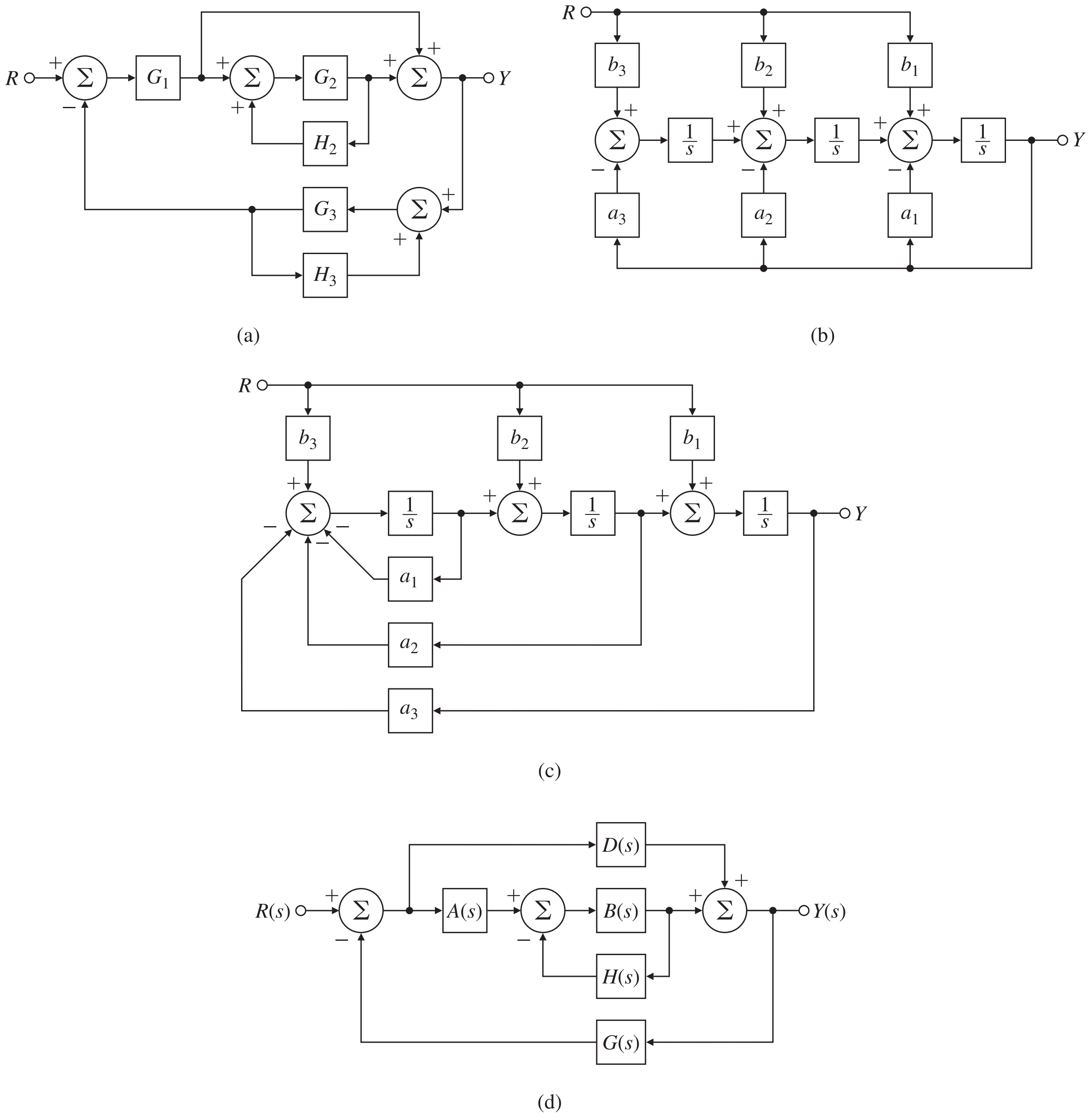


Figure 3.51
Block diagrams for Problem 3.21

3.26 For the unity feedback system shown in Fig. 3.54, specify the gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more than 10% in response to a unit step.

3.27 For the unity feedback system shown in Fig. 3.55, specify the gain and pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 30%, and a 2% settling time of no more than 0.2 sec. Verify your design using Matlab.

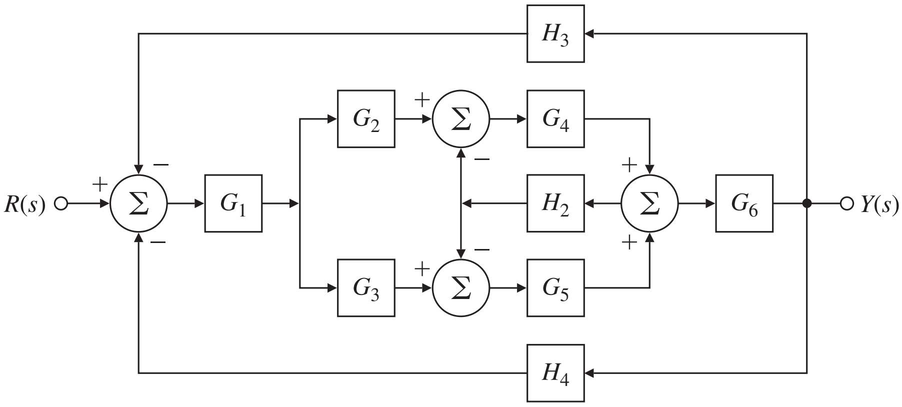


Figure 3.52
Block diagram for Problem 3.22

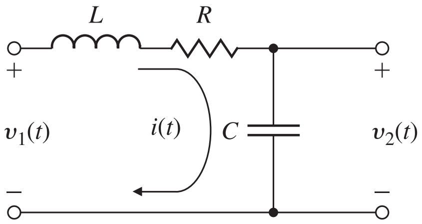


Figure 3.53
Circuit for Problem 3.25

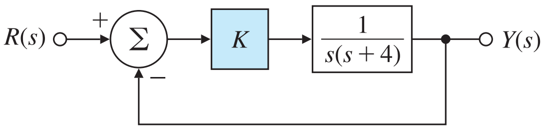


Figure 3.54
Unity feedback system for Problem 3.26

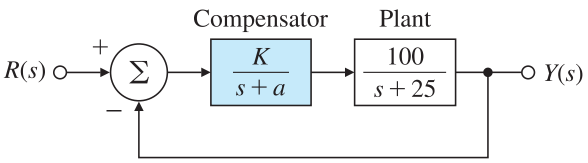


Figure 3.55
Unity feedback system for Problem 3.27

Problems for Section 3.4: Time-Domain Specification

3.28 A negative feedback control system is characterized by an open-loop transfer function

$$G(s)H(s) = \frac{k}{s(1+Ts)}.$$

Find, by what factor, the gain k of the system must be reduced so that the peak overshoot for unit-step response reduces from 75% to 25%.

3.29 A certain servomechanism has the dynamics dominated by a pair of complex poles at $s = (1 \pm j)$ and no zeros. Calculate the following time-domain specifications of this system:

(a) The damping ratio and natural frequency of this system

(b) Rise time

(c) Peak time

(d) Peak overshoot

(e) Settling time

(f) Time to 1st undershoot

(g) Time-domain expression for unit-step response

3.30 A unity negative feedback control system has a pair of real poles at $s = -2, -4$ and no zeros. Calculate the following:

(a) The damping ratio and natural frequency of this system

(b) Rise time

(c) Peak time

(d) Peak overshoot

(e) Settling time

(f) Time to 1st undershoot

(g) Time domain expression for unit step response

3.31 Suppose you are to design a unity feedback controller for a first-order plant depicted in Fig. 3.56. (As you will learn in Chapter 4, the configuration shown is referred to as a proportional–integral controller.) You are to design the controller so that the closed-loop poles lie within the shaded regions shown in Fig. 3.57.

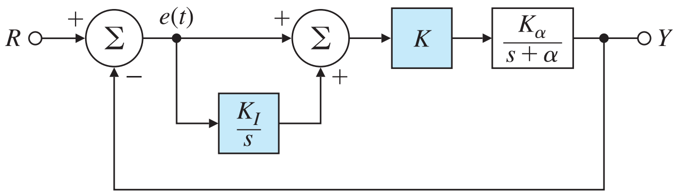


Figure 3.56
Unity feedback system for Problem 3.31

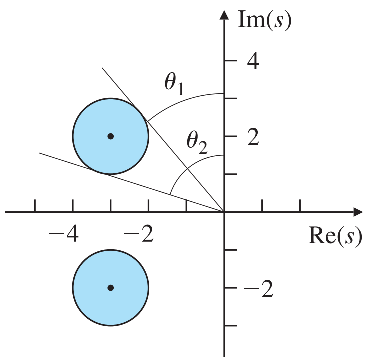


Figure 3.57
Desired closed-loop pole locations for Problem 3.31

(a) What values of ω_n and ζ correspond to the shaded regions in Fig. 3.57? (A simple estimate from the figure is sufficient.)

(b) Let $K_\alpha = \alpha = 2$. Find values for K and K_I so that the poles of the closed-loop system lie within the shaded regions.

(c) Prove that no matter what the values of K_α and α are, the controller provides enough flexibility to place the poles anywhere in the complex (left-half) plane.

3.32 The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s+5)}.$$

The desired system response to a step input is specified as peak time $t_p = 2$ sec and overshoot $M_p = 10\%$.

4 Erste Analyse des Regelkreises

Aufgabe 4.1: Review-Fragen

[FPE15, Seite 238]

1. Was ist das wichtigste Grund einen I-Anteil im Regler zu verwenden?
2. Was ist der wichtigste Grund einen D-Anteil im Regler zu verwenden
3. Warum sollte der D-Anteil in der Rückführung anstatt im Vorwärtszweig sein?

Aufgabe 4.2: Reglerauslegungen

Lösen Sie folgende Aufgaben aus [FPE15]:

4.24 (a), (b), (c) nur die Parameter(k, k_p, k_I) berechnen

4.27 (a), (b), (c)

4.29 (a)

4.30 (a), (b)

4.32 (a), (b)

4.34 alle Teilaufgaben (a) - (g)

4.35 (a), (b), (c)

Aufgabe 4.3: Routh'sches Stabilitätskriterium

Lösen Sie alle Aufgaben 3.53 bis 3.60.

Errata

3.57 Setzen Sie $K_o = 1$

4.23 Consider the system shown in Fig. 4.38.

(a) Find the transfer function from the reference input to the tracking error.

(b) For this system to respond to inputs of the form $r(t) = t^n 1(t)$ (where $n < q$) with zero steady-state error, what constraint is placed on the open-loop poles $p_1, p_2, \cdots, p_q$?

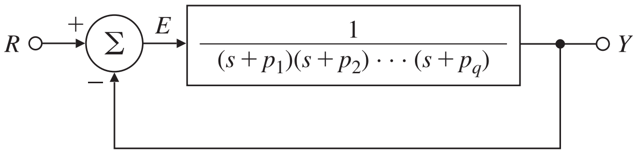


Figure 4.38
Control system for Problem 4.23

4.24 Consider the system shown in Fig. 4.39.

(a) Compute the transfer function from $R(s)$ to $E(s)$ and determine the steady-state error (e_{ss}) for a unit-step reference input signal, and a unit-ramp reference input signal.

(b) Determine the locations of the closed-loop poles of the system.

(c) Select the system parameters (k, k_P, k_I) such that the closed-loop system has damping coefficient $\zeta = 0.707$ and $\omega_n = 1$. What percent overshoot would you expect in $y(t)$ for unit-step reference input?

(d) Find the tracking error signal as a function of time, $e(t)$, if the reference input to the system, $r(t)$, is a unit-ramp.

(e) How can we select the PI controller parameters (k_P, k_I) to ensure that the amplitude of the transient tracking error, $|e(t)|$, from part (d) is small?

(f) What is the transient behavior of the tracking error, $e(t)$, for a unit-ramp reference input if the magnitude of the integral gain, k_I, is very large? Does the unit-ramp response have an overshoot in that case?

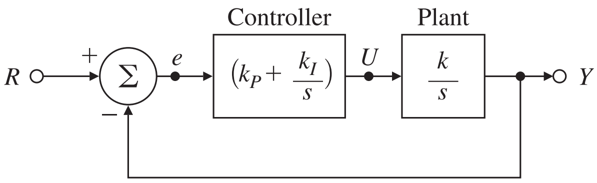


Figure 4.39
Control system diagram for Problem 4.24

4.25 A linear ODE model of the DC motor with negligible armature inductance ($L_a = 0$) and with a disturbance torque w was given earlier in the chapter; it is restated here, in slightly different form, as

$$\frac{JR_a}{K_t}\ddot{\theta}_m + K_e\dot{\theta}_m = \upsilon_a + \frac{R_a}{K_t}w,$$

where θ_m is measured in radians. Dividing through by the coefficient of $\ddot{\theta}_m$, we obtain

$$\ddot{\theta}_m + a_1\dot{\theta}_m = b_0\upsilon_a + c_0 w,$$

where

$$a_1 = \frac{K_t K_e}{JR_a}, \quad b_0 = \frac{K_t}{JR_a}, \quad c_0 = \frac{1}{J}.$$

With rotating potentiometers, it is possible to measure the positioning error between θ and the reference angle θ_r or $e = \theta_{ref} - \theta_m$. With a tachometer we can measure the motor speed $\dot{\theta}_m$. Consider using feedback of the error e and the motor speed $\dot{\theta}_m$ in the form

$$\upsilon_a = K(e - T_D\dot{\theta}_m),$$

where K and T_D are controller gains to be determined.

(a) Draw a block diagram of the resulting feedback system showing both θ_m and $\dot{\theta}_m$ as variables in the diagram representing the motor.

(b) Suppose the numbers work out so that $a_1 = 65$, $b_0 = 200$, and $c_0 = 10$. If there is no load torque ($w = 0$), what speed (in rpm) results from $v_a = 100$ V?

(c) Using the parameter values given in part (b), let the control be $D = k_P + k_D s$ and find k_P and k_D so that, using the results of Chapter 3, a step change in θ_{ref} with zero load torque results in a transient that has an approximately 17% overshoot and that settles to within 5% of steady state in less than 0.05 sec.

(d) Derive an expression for the steady-state error to a reference angle input and compute its value for your design in part (c) assuming $\theta_{ref} = 1$ rad.

(e) Derive an expression for the steady-state error to a constant disturbance torque when $\theta_{ref} = 0$ and compute its value for your design in part (c) assuming $w = 1.0$.

4.26 We wish to design an automatic speed control for an automobile. Assume that (1) the car has a mass m of 1000 kg, (2) the accelerator is the control U and supplies a force on the automobile of 10 N per degree of accelerator motion, and (3) air drag provides a friction force proportional to velocity of $10 \text{ N} \cdot \text{sec/m}$.

(a) Obtain the transfer function from control input U to the velocity of the automobile.

(b) Assume the velocity changes are given by

$$V(s) = \frac{1}{s + 0.02}U(s) + \frac{0.05}{s + 0.02}W(s),$$

where V is given in meters per second, U is in degrees, and W is the percent grade of the road. Design a proportional control law $U = -k_P V$ that will maintain a velocity error of less than 1 m/sec in the presence of a constant 2% grade.

(c) Discuss what advantage (if any) integral control would have for this problem.

(d) Assuming that pure integral control (that is, no proportional term) is advantageous, select the feedback gain so that the roots have critical damping ($\zeta = 1$).

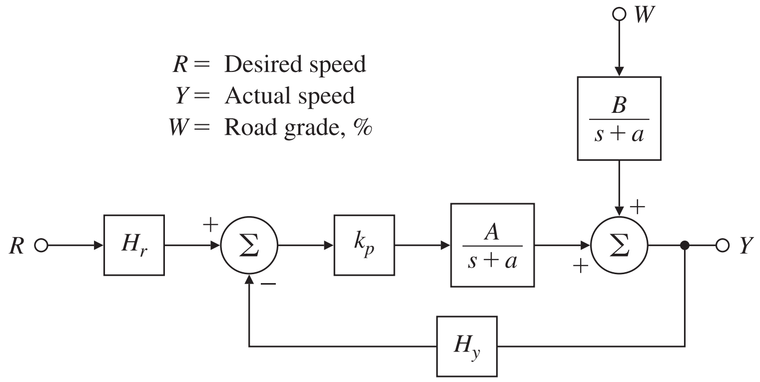
4.27 Consider the automobile speed control system depicted in Fig. 4.40.

(a) Find the transfer functions from $W(s)$ and from $R(s)$ to $Y(s)$.

(b) Assume that the desired speed is a constant reference r, so that $R(s) = \frac{r_o}{s}$. Assume that the road is level, so $w(t) = 0$. Compute values of the gains k_P, H_r, and H_y to guarantee that

$$\lim_{t\to\infty} y(t) = r_o.$$

Figure 4.40
Automobile speed-control system



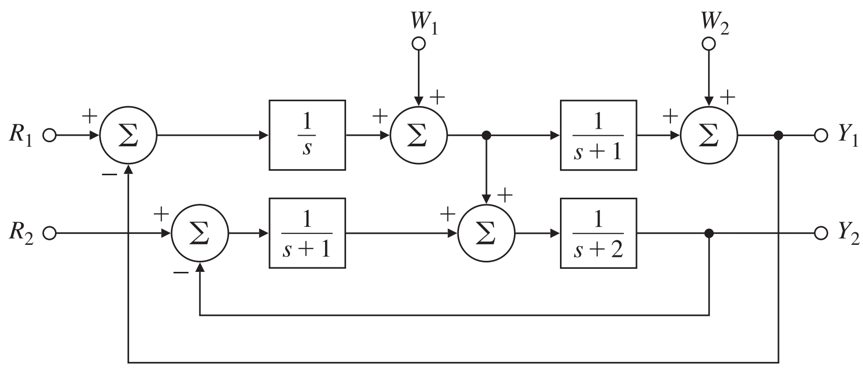
Include both the open-loop (assuming $H_y = 0$) and feedback cases ($H_y \neq 0$) in your discussion.

(c) Repeat part (b) assuming that a constant grade disturbance $W(s) = \frac{w_o}{s}$ is present *in addition to* the reference input. In particular, find the variation in speed due to the grade change for both the feed forward and feedback cases. Use your results to explain (1) why feedback control is necessary and (2) how the gain k_P should be chosen to reduce steady-state error.

(d) Assume that $w(t) = 0$ and that the gain A undergoes the perturbation $A + \delta A$. Determine the error in speed due to the gain change for both the feed forward and feedback cases. How should the gains be chosen in this case to reduce the effects of δA?

4.28 Consider the multivariable system shown in Fig. 4.41. Assume that the system is stable. Find the transfer functions from each disturbance input to each output and determine the steady-state values of y_1 and y_2 for constant disturbances. We define a multivariable system to be type k with respect to polynomial inputs at w_i if the steady-state value of *every* output is zero for any combination of inputs of degree less than k and at least one input is a nonzero constant for an input of degree k. What is the system type with respect to disturbance rejection at w_1 and at w_2?

Figure 4.41
Multivariable system



Problems for Section 4.3: The Three-Term Controller: PID Control

4.29 The transfer functions of speed control for a magnetic tape-drive system are shown in Fig. 4.42. The speed sensor is fast enough that its dynamics can be neglected and the diagram shows the equivalent unity feedback system.

(a) Assuming the reference is zero, what is the steady-state error due to a step disturbance torque of 1 N·m? What must the amplifier gain K be in order to make the steady-state error $e_{ss} \leq 0.01$ rad/sec?

(b) Plot the roots of the closed-loop system in the complex plane and accurately sketch the time response of the output for a step reference input using the gain K computed in part (a).

(c) Plot the region in the complex plane of acceptable closed-loop poles corresponding to the specifications of a 1% settling time of $t_s \leq 0.1$ sec and an overshoot $M_p \leq 5\%$.

(d) Give values for k_P and k_D for a PD controller, which will meet the specifications.

(e) How would the disturbance-induced steady-state error change with the new control scheme in part (d)? How could the steady-state error to a disturbance torque be eliminated entirely?

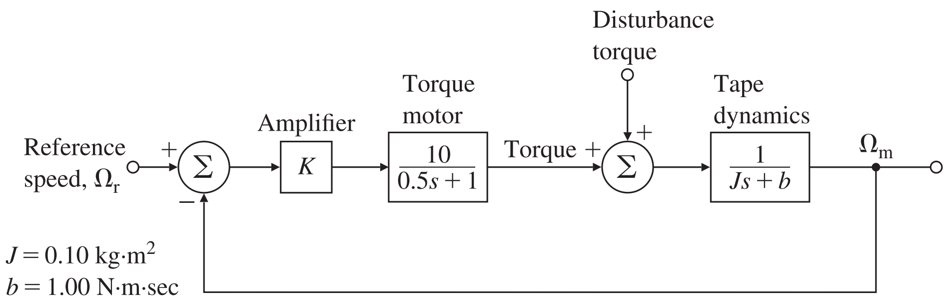


Figure 4.42
Speed-control system for a magnetic tape drive

4.30 Consider the system shown in Fig. 4.43 with PI control.

(a) Determine the transfer function from R to Y.

(b) Determine the transfer function from W to Y.

(c) What are the system type and error constant with respect to reference tracking?

(d) What are the system type and error constant with respect to disturbance rejection?

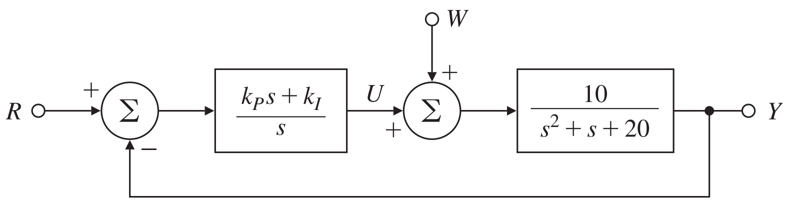


Figure 4.43
Control system for Problem 4.30

4.31 Consider the second-order plant with transfer function

$$G(s) = \frac{1}{(s+1)(5s+1)},$$

and in a unity feedback structure.

(a) Determine the system type and error constant with respect to tracking polynomial reference inputs of the system for P [$D_c = k_P$], PD [$D_c(s) = k_P + k_D s$], and PID [$D_c(s) = k_P + \frac{k_I}{s} + k_D s$] controllers. Let $k_P = 19$, $k_I = 0.5$, and $k_D = \frac{4}{19}$.

(b) Determine the system type and error constant of the system with respect to disturbance inputs for each of the three regulators in part (a) with respect to rejecting polynomial disturbances $w(t)$ at the *input* to the plant.

(c) Is this system better at tracking references or rejecting disturbances? Explain your response briefly.

(d) Verify your results for parts (a) and (b) using Matlab by plotting unit-step and -ramp responses for both tracking and disturbance rejection.

4.32 The DC motor speed control shown in Fig. 4.44 is described by the differential equation

$$\dot{y} + 60y = 600v_a - 1500w,$$

where y is the motor speed, v_a is the armature voltage, and w is the load torque. Assume the armature voltage is computed using the PI control law

$$v_a = -\left(k_P e + k_I \int_0^t e\,dt\right),$$

where $e = r - y$.

(a) Compute the transfer function from W to Y as a function of k_P and k_I.

(b) Compute values for k_P and k_I so that the characteristic equation of the closed-loop system will have roots at $-60 \pm 60j$.

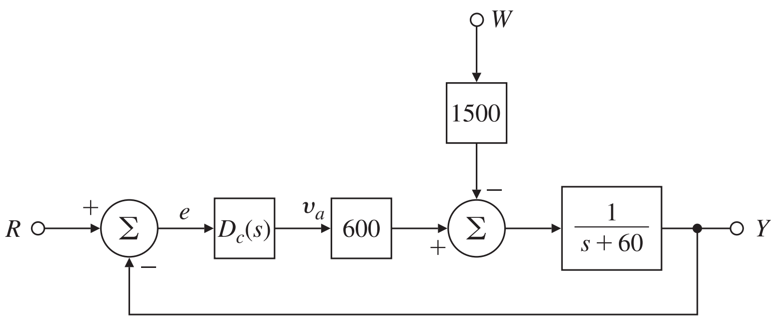


Figure 4.44
DC Motor speed-control block diagram for Problems 4.32 and 4.33

4.33 For the system in Fig. 4.44, compute the following steady-state errors:

(a) to a unit-step reference input;

(b) to a unit-ramp reference input;

(c) to a unit-step disturbance input;

(d) for a unit-ramp disturbance input.

(e) Verify your answers to (a) and (d) using Matlab. Note that a ramp response can be generated as a step response of a system modified by an added integrator at the reference input.

4.34 Consider the satellite-attitude control problem shown in Fig. 4.45 where the normalized parameters are

$$J = 10 \quad \text{spacecraft inertia, N·m·sec}^2/\text{rad}$$
$$\theta_r = \text{reference satellite attitude, rad.}$$
$$\theta = \text{actual satellite attitude, rad.}$$
$$H_y = 1 \quad \text{sensor scale, factor V/rad.}$$
$$H_r = 1 \quad \text{reference sensor scale factor, V/rad.}$$
$$w = \text{disturbance torque, N·m.}$$

(a) Use proportional control, P, with $D_c(s) = k_P$, and give the range of values for k_P for which the system will be stable.

(b) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to reference inputs.

(c) Use PD control, let $D_c(s) = (k_P + k_D s)$, and determine the system type and error constant with respect to disturbance inputs.

(d) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to reference inputs.

(e) Use PI control, let $D_c(s) = (k_P + \frac{k_I}{s})$, and determine the system type and error constant with respect to disturbance inputs.

(f) Use PID control, let $D_c(s) = (k_P + \frac{k_I}{s} + k_D s)$, and determine the system type and error constant with respect to reference inputs.

(g) Use PID control, let $D_c(s) = (k_P + \frac{k_I}{s} + k_D s)$, and determine the system type and error constant with respect to disturbance inputs.

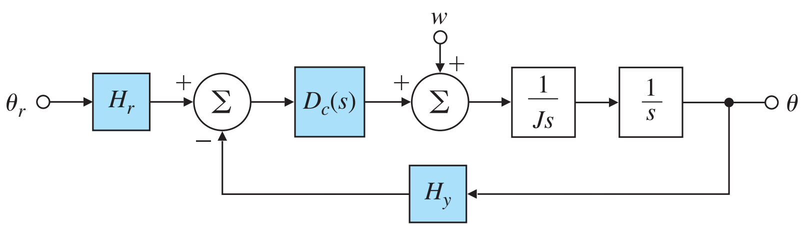


Figure 4.45
Satellite attitude control

4.35 Automatic ship steering is particularly useful in heavy seas when it is important to maintain the ship along an accurate path. Such a control system for a large tanker is shown in Fig. 4.46, with the plant transfer function relating heading changes to rudder deflection in radians.

(a) Write the differential equation that relates the heading angle to rudder angle for the ship *without* feedback.

(b) This control system uses simple proportional feedback with the gain of unity. Is the closed-loop system stable as shown? (*Hint:* use Routh's criterion.)

(c) Is it possible to stabilize this system by changing the proportional gain from unity to a lower value?

(d) Use Matlab to design a dynamic controller of the form $D_c(s) = K\left(\frac{s+a}{s+b}\right)^2$ so that the closed-loop system is stable and in response to a step heading command it has zero steady-state error and less than 10% overshoot. Are these reasonable values for a large tanker?

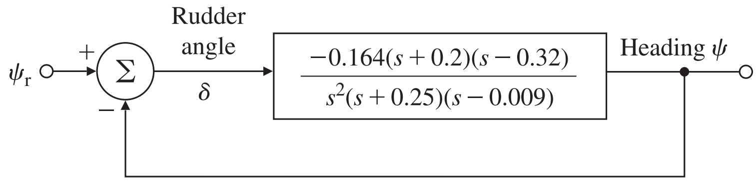


Figure 4.46
Ship-steering control system for Problem 4.35

4.36 The unit-step response of a paper machine is shown in Fig. 4.47(a) where the input into the system is stock flow onto the wire and the output is basis weight (thickness). The time delay and slope of the transient response may be determined from the figure.

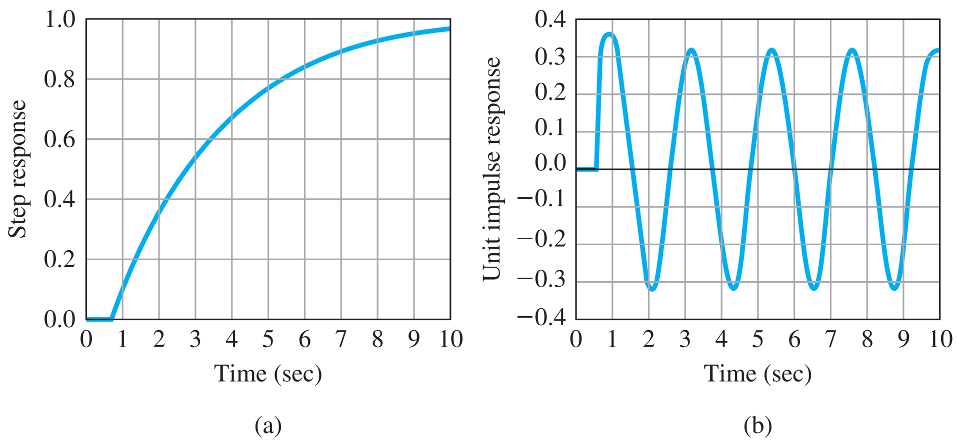


Figure 4.47
Paper-machine response data for Problem 4.36

(a) Find the proportional-, PI-, and PID-controller parameters using the Ziegler–Nichols transient-response method.

(b) Using proportional feedback control, control designers have obtained a closed-loop system with the unit impulse response shown in Fig. 4.47(b). When the gain $K_u = 8.556$, the system is on the verge of instability. Determine the proportional-, PI-, and PID-controller parameters according to the Ziegler–Nichols ultimate sensitivity method.

4.37 A paper machine has the transfer function

$$G(s) = \frac{e^{-2s}}{3s+1},$$

where the input is stock flow onto the wire and the output is basis weight or thickness.

(a) Find the PID-controller parameters using the Ziegler–Nichols tuning rules.

(b) The system becomes marginally stable for a proportional gain of $K_u = 3.044$ as shown by the unit impulse response in Fig. 4.48. Find the optimal PID-controller parameters according to the Ziegler–Nichols tuning rules.

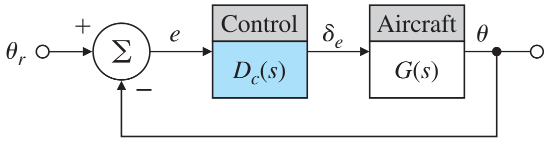


Figure 3.64
Block diagram of autopilot for Problem 3.51

Problems for Section 3.6: Stability

3.52 A measure of the degree of instability in an unstable aircraft response is the amount of time it takes for the *amplitude* of the time response to double (see Fig. 3.65), given some nonzero initial condition.

(a) For a first-order system, show that the **time to double** is

$$\tau_2 = \frac{\ln 2}{p}$$

where p is the pole location in the RHP.

(b) For a second-order system (with two complex poles in the RHP), show that

$$\tau_2 = \frac{\ln 2}{-\zeta\omega_n}.$$

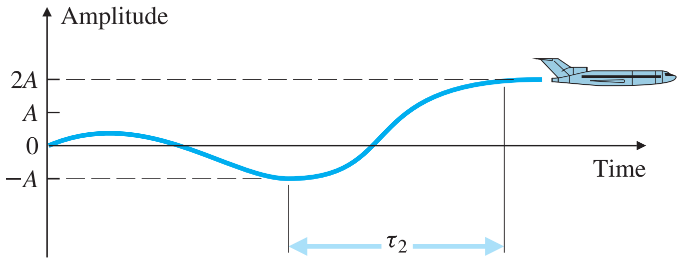


Figure 3.65
Time to double

3.53 For the given open-loop systems, considering a unity feedback case, determine their closed-loop stability.

(a) $G(s)H(s) = \dfrac{s+4}{s(s+3)(s+8)}$

(b) $G(s)H(s) = \dfrac{2(s+1)}{s^3+s^2+2s+1}$

(c) $G(s)H(s) = \dfrac{10}{(s+2)(s+4)(s^2+6s+25)}$

3.54 Using Routh's stability criterion, determine the stability of the systems with the following characteristic equations:

(a) $s^4+8s^3+18s^2+16s+5=0$

(b) $3s^4+10s^3+5s^2+5s+2=0$

(c) $s^5+s^4+2s^3+2s^2+3s+5=0$

(d) $s^3+7s^2+25s+39=0$

(e) $s^6+s^5+3s^4+3s^3+3s^2+2s+1=0$

3.55 A positional servomechanism is characterized by an open-loop transfer function

$$G(s)H(s) = \frac{k(s+2)}{s(s-1)}.$$

Determine the value of the gain k when (a) the damping ratio is 0.5 and (b) the closed-loop system has two roots on the $j\omega$ axis.

3.56 The unity feedback system is given by

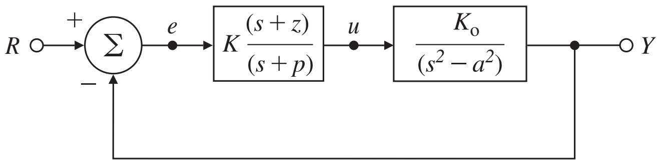
$$G(s)H(s) = \frac{k(s+4)}{s(s+1)(s+2)}.$$

(a) Find the range of k that makes the system stable.

(b) Find the value of k that makes the system oscillate.

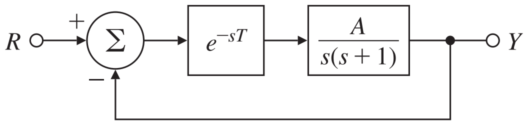
3.57 Consider the closed-loop magnetic levitation system shown in Fig. 3.66. Determine the conditions on the system parameters (a, K, z, p, K_o) to guarantee closed-loop system stability.

Figure 3.66
Magnetic levitation system for Problem 3.57



3.58 Consider the system shown in Fig. 3.67.

Figure 3.67
Control system for Problem 3.58



(a) Compute the closed-loop characteristic equation.

(b) For what values of (T, A) is the system stable? *Hint*: An approximate answer may be found using

$$e^{-Ts} \cong 1 - Ts,$$

or

$$e^{-Ts} \cong \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s},$$

for the pure delay. As an alternative, you could use the computer Matlab (Simulink) to simulate the system or to find the roots of the system's characteristic equation for various values of T and A.

3.59 A negative feedback system is represented by the following equation

$$F(s) = s^3 + 10s^2 + 29s + k.$$

Shift the vertical axis of the s-plane to the right by two units using the condition $s = s_1 - 2$ and determine the value of the gain k so that the complex roots are at $s = -2 \pm j$.

3.60 A unity feedback system is given by

$$G(s)H(s) = \frac{k(s+1)}{s^3 + \alpha s^2 + 2s + 1}.$$

5 Wurzelortskurve

Aufgabe 5.1: Review-Fragen

1. Geben Sie zwei Definitionen für die Wurzelortskurve.
2. Wo liegen die Abschnitte der WOK auf der reellen Achse?
3. Was sind die Startwinkel der WOK in einem doppelten Pol bei $s = -a$ auf der reellen Achse? Nehmen Sie an, dass es keine Pole oder Nullstellen rechts von $-a$ gibt.
4. Warum ist der Start-Winkel von einem Pol in der Nähe der imaginären Achse besonders wichtig?

Aufgabe 5.2: Wurzelortskurven

Lösen Sie alle Aufgaben 5.1 bis 5.14 aus [FPE15]:

5.9 What is meant by "k" in a root locus? How can one find the value of k mathematically?

5.10 Define a marginally stable system.

5.11 Show, with a root-locus argument, that a system having three poles at the origin MUST be either unstable or, at best, conditionally stable.

PROBLEMS

Problems for Section 5.1: Root Locus of a Basic Feedback System

5.1 Set up the listed characteristic equations in the form suited to Evans's root-locus method. Give $L(s)$, $a(s)$, and $b(s)$ and the parameter K in terms of the original parameters in each case. Be sure to select K so that $a(s)$ and $b(s)$ are monic in each case and the degree of $b(s)$ is not greater than that of $a(s)$.

(a) $s + (1/\tau) = 0$ versus parameter τ

(b) $s^2 + cs + c + 1 = 0$ versus parameter c

(c) $(s + c)^3 + A(Ts + 1) = 0$

(i) versus parameter A,

(ii) versus parameter T,

(iii) versus the parameter c, if possible. Say why you can or cannot. Can a plot of the roots be drawn versus c for given constant values of A and T by any means at all?

(d) $1 + \left[k_p + k_I(s) + \frac{k_D s}{\tau s+1}\right] G(s) = 0$. Assume that $G(s) = A\frac{c(s)}{d(s)}$, where $c(s)$ and $d(s)$ are monic polynomials with the degree of $d(s)$ greater than that of $c(s)$.

(i) versus k_p

(ii) versus k_I

(iii) versus k_D

(iv) versus τ

Problems for Section 5.2: Guidelines for Sketching a Root Locus

5.2 Roughly sketch the root loci for the pole–zero maps as shown in Fig. 5.44 without the aid of a computer. Show your estimates of the center and angles of the asymptotes, a rough evaluation of arrival and departure angles for complex poles and zeros, and the loci for positive values of the parameter K. Each pole–zero map is from a characteristic equation of the form

$$1 + K\frac{b(s)}{a(s)} = 0,$$

where the roots of the numerator $b(s)$ are shown as small circles $\circ$ and the roots of the denominator $a(s)$ are shown as $\times$'s on the s-plane. Note that in Fig. 5.44(c) there are two poles at the origin.

5.3 For the open-loop transfer function

$$G(s)H(s) = \frac{k}{s(s+3)(s^2+2s+2)},$$

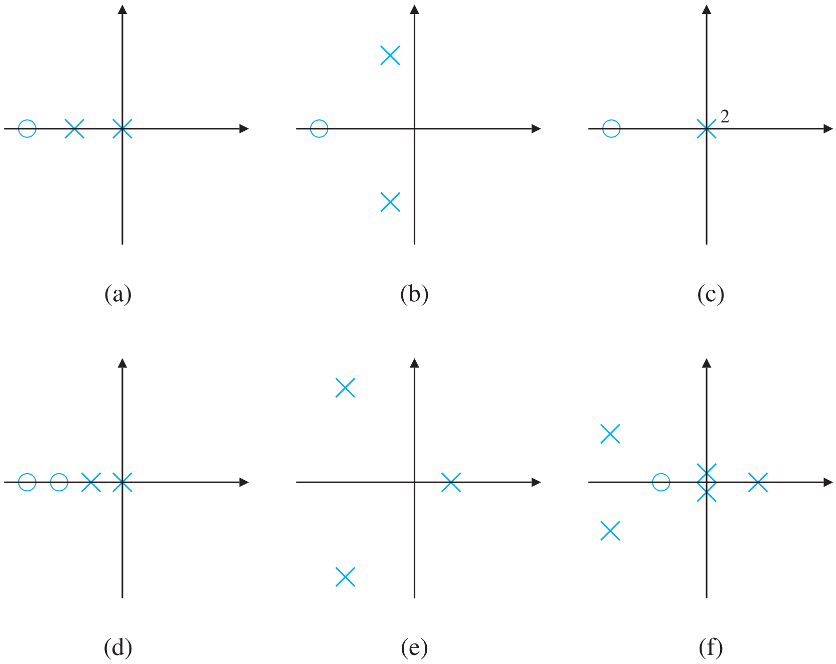


Figure 5.44
Pole–zero maps

(a) Draw the pole-zero plot.

(b) Find the asymptotes for $K \to \infty$.

(c) Sketch the root locus.

5.4 *Real poles and zeros.* For the systems with an open-loop transfer function given below, sketch the root locus plot. Find the asymptotes and their angles, the break-away or break-in points, the angle of arrival or departure for the complex poles and zeros, respectively, and the range of k for closed-loop stability.

(a) $G(s)H(s) = \frac{k}{s(s+1)(s+2)(s+3)}$

(b) $G(s)H(s) = \frac{k(s+1)}{s(s-1)}$

(c) $G(s)H(s) = \frac{k}{(s+1)(s+2)(s+4)}$

(d) $G(s)H(s) = \frac{k(s+1)(s+2)}{s(s-1)}$

5.5 *Complex poles and zeros.* For the systems with an open-loop transfer function given below, sketch the root locus plot. Find the asymptotes and their angles, the break-away or break-in points, the angle of arrival or departure for the complex poles and zeros, respectively, and the range of k for closed-loop stability.

(a) $G(s)H(s) = \frac{k}{s(s^2+6s+12)}$

(b) $G(s)H(s) = \frac{k}{(s+1)(s+2+j)(s+2-j)}$

(c) $G(s)H(s) = \frac{k(s+2)}{(s+1)(s^2+6s+8)}$

(d) $G(s)H(s) = \frac{k}{s(s+3)(s^2+2s+2)}$

(e) $G(s)H(s) = \frac{ks}{(s^2-s+4.25)}$

(f) $G(s)H(s) = \frac{k(s^2-4s+8)}{(s+1)(s-0.5)}$

5.6 *Multiple poles at the origin.* For the systems with an open-loop transfer function given below, sketch the root locus plot. Find the asymptotes and their angles, the break-away or break-in points, the angle of arrival or departure for the complex poles and zeros, respectively, and the range of k for closed-loop stability.

(a) $G(s)H(s) = \frac{k}{s(s+1)(s+2)}$

(b) $G(s)H(s) = \frac{k(s+1)}{s^2(s+9)}$

(c) $G(s)H(s) = \frac{k(s+4)}{s(s+1)}$

(d) $G(s)H(s) = \frac{k}{s^2(s+2)}$

(e) $G(s)H(s) = \frac{k(s+4)}{s^2(s+1)}$

(f) $G(s)H(s) = \frac{k(s+4)}{s^3(s+1)}$

(g) $G(s)H(s) = \frac{ks^2}{(s+1)(s+2)}$

5.7 *Mixed real and complex poles.* For the systems with an open-loop transfer function given below, sketch the root locus plot. Find the asymptotes and their angles, the break-away or break-in points, the angle of arrival or departure for the complex poles and zeros, respectively, and the range of k for closed-loop stability.

(a) $G(s)H(s) = \frac{k}{s(s^2+6s+10)}$

(b) $G(s)H(s) = \frac{k}{s(s+4)(s^2+4s+20)}$

(c) $G(s)H(s) = \frac{k(s+1)}{s^4+2s^3+12s^2+16s}$

(d) $G(s)H(s) = \frac{k(s+2)}{s(s^2+2s+2)}$

(e) $G(s)H(s) = \frac{k}{s(s+1)^2(s+2)}$

5.8 *RHP and zeros*. For the systems with an open-loop transfer function given below, sketch the root locus plot. Find the asymptotes and their angles, the break-away or break-in points, and the angle of arrival or departure for the complex poles and zeros, respectively, if any.

(a) $G(s)H(s) = \frac{s+1}{s+3}\frac{1}{s^2-2}$; the model for the case of a temperature controller with lead compensation.

(b) $G(s)H(s) = \frac{s+1}{s(s+3)}\frac{1}{s^2-2}$; the model for the case of a temperature controller with integral control and lead compensation.

(c) $G(s)H(s) = \frac{s-2}{s^2}$

(d) $G(s)H(s) = \frac{(s^2-2)(s+4)}{s(s+2)^2(s+3)^2}$

(e) $G(s)H(s) = \frac{s+3}{s(s-2)(s-3)^2}$

(f) $G(s)H(s) = \frac{1}{s(s-2)(s-3)^2}$

(*Note: The above systems are to be considered to have a unity feedback.*)

5.9 For the system shown in Fig. 5.45, find the root locus plot for all values of k ranging from zero to infinity. Find the asymptotes and their angles, angle of arrival and departure for complex poles and zeros, if any, and the break-away points. Also find the gain k for which the closed-loop transfer function will have a pole on the real axis at -0.5.

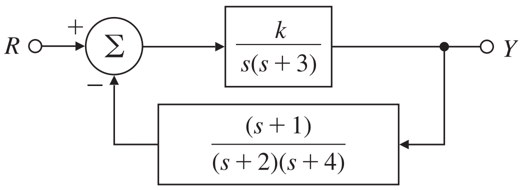


Figure 5.45
Control system for Problem 5.9

5.10 For the system with an open-loop transfer function given below

$$G(s)H(s) = \frac{k(s^2 - 4s + 20)}{(s+2)(s+4)}$$

(a) Find the gain k at the $j\omega$ crossing.

(b) Find the gain at the point where the root locus crosses the 0.45 damping ratio line.

5.11 Use Routh criterion to find the range of K for which the systems in Fig. 5.46 are stable. Use the root locus technique to confirm your calculation.

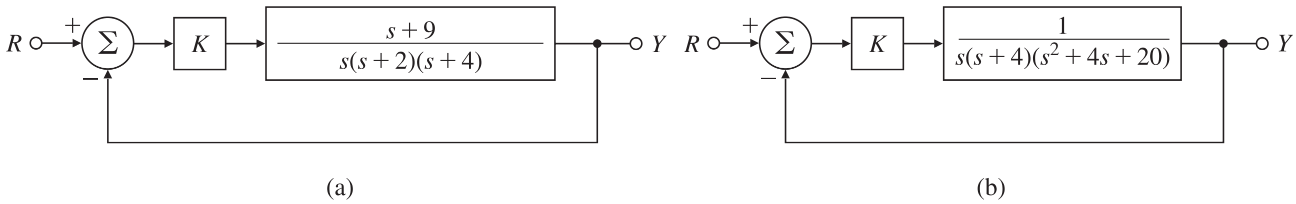


Figure 5.46
Feedback systems for Problem 5.11

5.12 Sketch the root locus plot for a negative feedback control system given below

$$G(s)H(s) = \frac{k}{(s+1)(s+2)(s+3)} \qquad 0 \le k \le \infty.$$

(a) Find the range of k for stability.

(b) Find k for 20% overshoot.

(c) For k found in (a), find ts and tp.

5.13 For the system shown in Fig. 5.47:

(a) Sketch the root locus plot.

(b) Find the range of k for stability.

(c) Find the value of k that causes oscillations.

(d) Find the value of k for a damping ratio of 0.707.

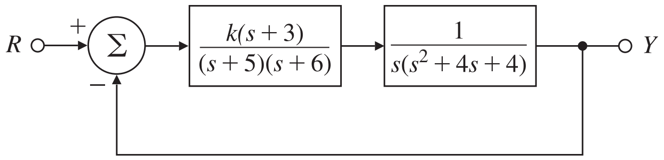


Figure 5.47
Feedback system for Problem 5.13

5.14 For the feedback system shown in Fig. 5.48, find the value of the gain K that results in dominant closed-loop poles with a damping ratio $\zeta = 0.5$.

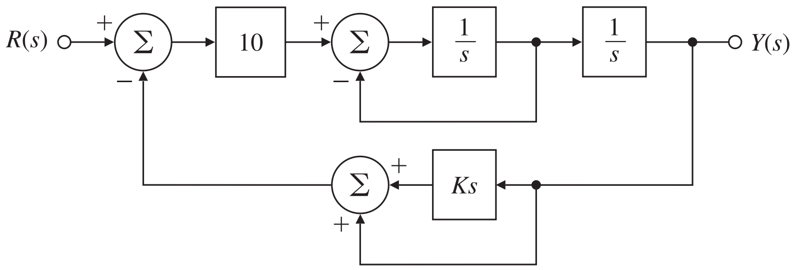


Figure 5.48
Feedback system for Problem 5.14

Problems for Section 5.3: Selected Illustrative Root Loci

5.15 A simplified model of the longitudinal motion of a certain helicopter near hover has the transfer function

$$G(s) = \frac{9.8(s^2 - 0.5s + 6.3)}{(s + 0.66)(s^2 - 0.24s + 0.15)}$$

and the characteristic equation $1 + D_c(s)G(s) = 0$. Let $D_c(s) = k_p$ at first.

(a) Compute the departure and arrival angles at the complex poles and zeros.

(b) Sketch the root locus for this system for parameter $K = 9.8k_p$. Use axes $-4 \le x \le 4$; $-3 \le y \le 3$.

(c) Verify your answer using Matlab. Use the command `axis([−4 4 −3 3])` to get the right scales.

(d) Suggest a practical (at least as many poles as zeros) alternative compensation $D_c(s)$ that will at least result in a stable system.

5.16 **(a)** For the system given in Fig. 5.49, plot the root locus of the characteristic equation as the parameter K_1 is varied from 0 to ∞ with $\lambda = 2$. Give the corresponding $L(s)$, $a(s)$, and $b(s)$.

(b) Repeat part (a) with $\lambda = 5$. Is there anything special about this value?

6 Bode-Diagramm

Aufgabe 6.1: Review-Fragen

1. Warum schlug Bode vor, den Amplitudengang einer Frequenzantwort doppelt-logarithmisch darzustellen?

2. Definieren Sie Dezibel.

3. Was ist die Amplitude der Übertragungsfunktion bei einer Verstärkung von 14 dB

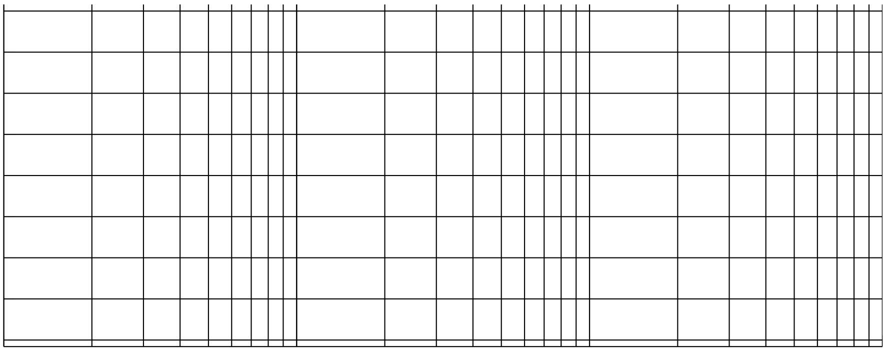
Aufgabe 6.2: [FPE15, 6.2]

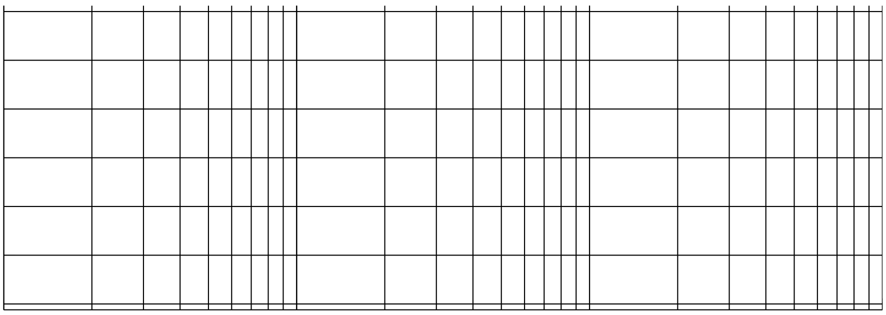
(a) Bestimmen Sie den Amplitude und Phase von

$$G(s) = \frac{1}{s+5}$$

von Hand für $\omega = 1, 2, 5, 10, 20, 50$ und $100\,\text{rad/sec}$.

(b) Skizzieren Sie die Asymptoten von $G(s)$ und vergleichen Sie diese mit ihren berechneten Ergebnissen aus Teil (a).

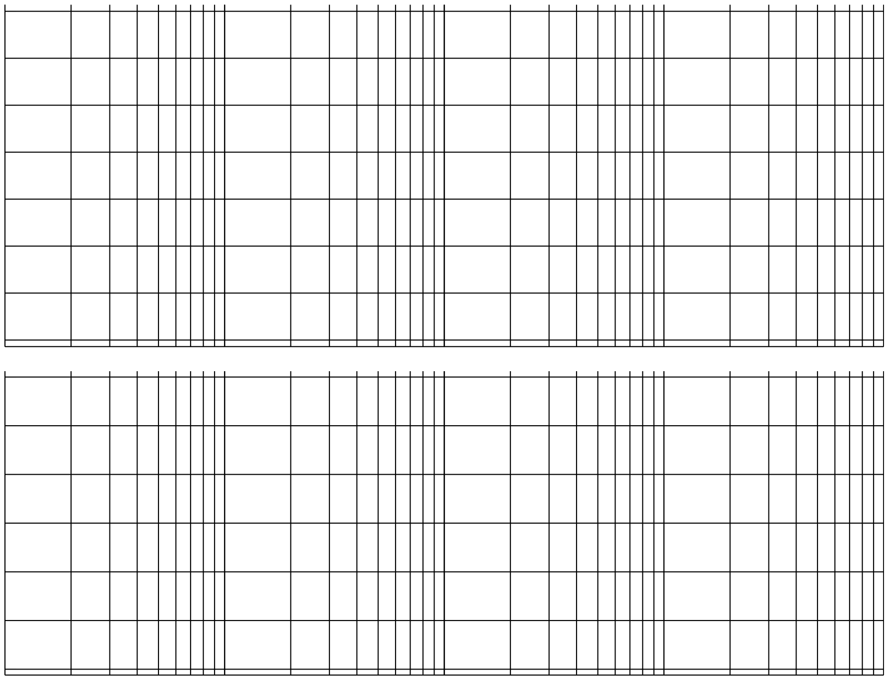




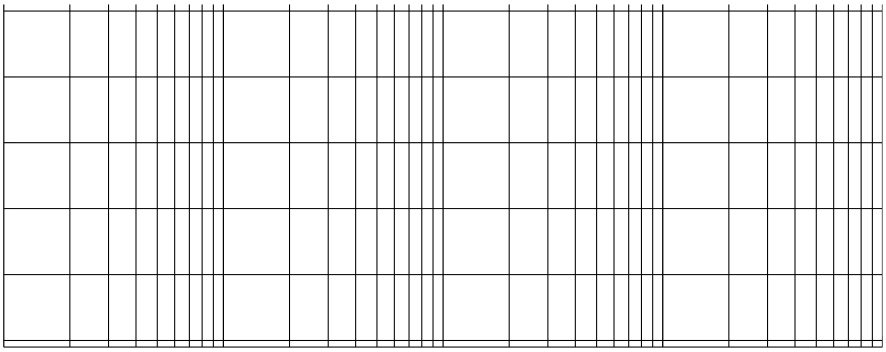
Aufgabe 6.3: [FPE15, 6.3]

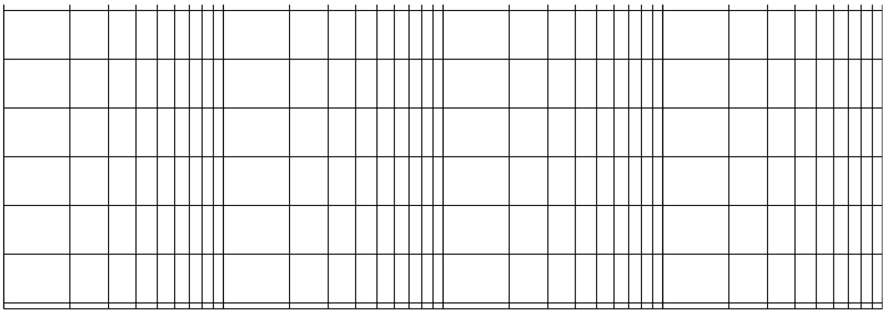
Skizzieren Sie die Amplituden- und Phasengänge (Asymptoten und ungefährer Verlauf) folgender Übertragungsfunktionen. Bestimmen Sie hierzu die Asymptoten, die Funktionswerte an den Eckfrequenzen und beschriften Sie die Achsen. Bestimmen Sie aus dem Diagramm die Betragsreserve, die Durchtrittskreis- sowie die Phasenschnittkreisfrequenz.

(a) $L(s) = \dfrac{10}{s\ (s+10)}$

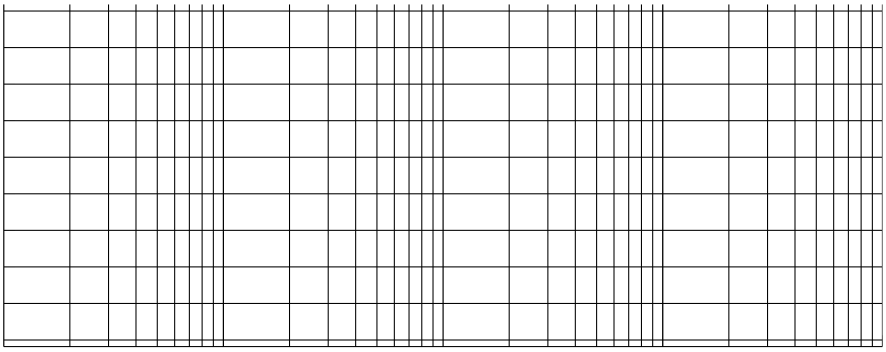


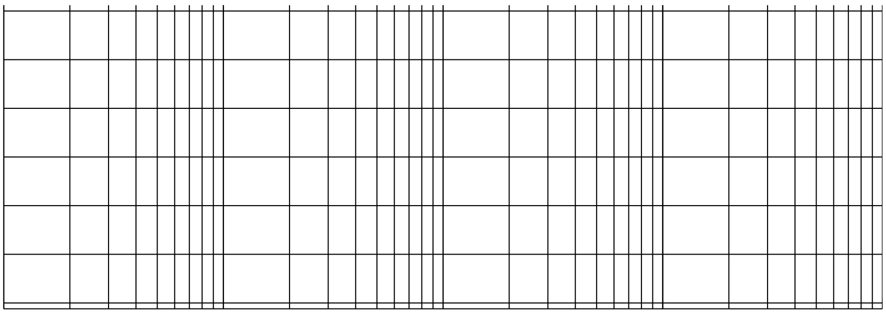
(b) $L(s) = \dfrac{200}{s\ (s+2)\ (s+10)}$



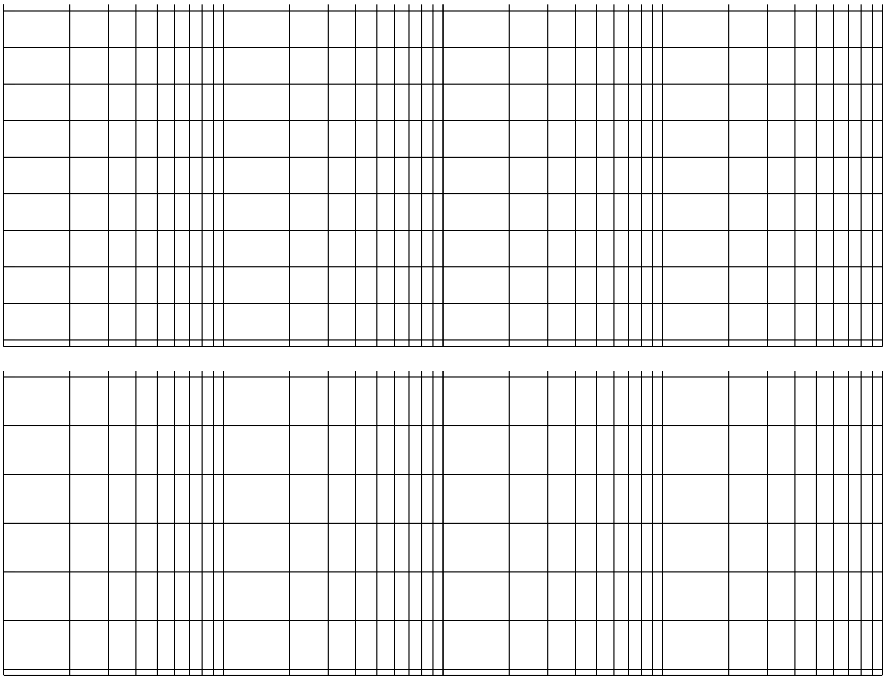


(c) $L(s) = \dfrac{5}{s\ (s+1)\ (s+2)}$

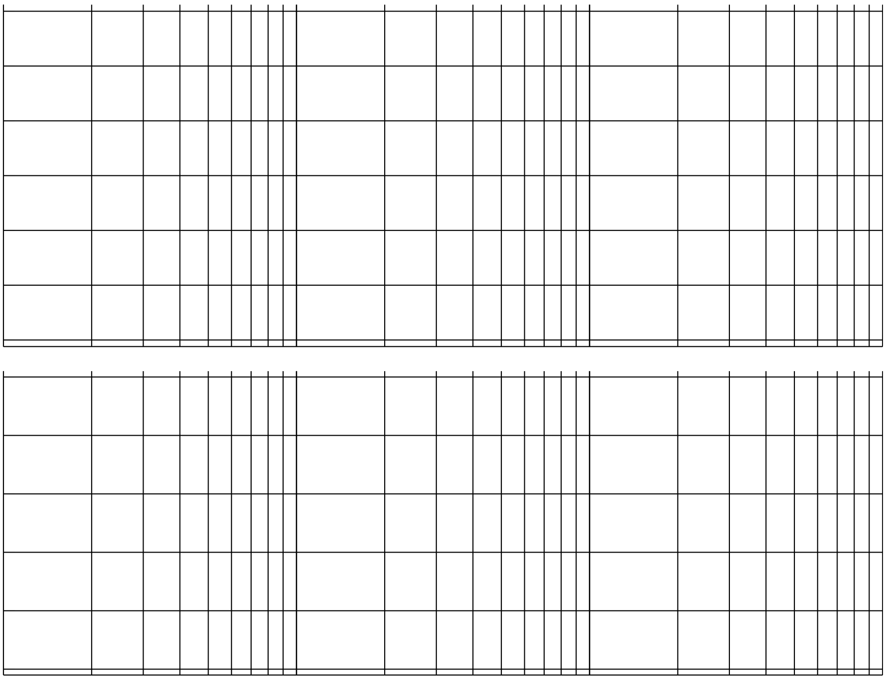




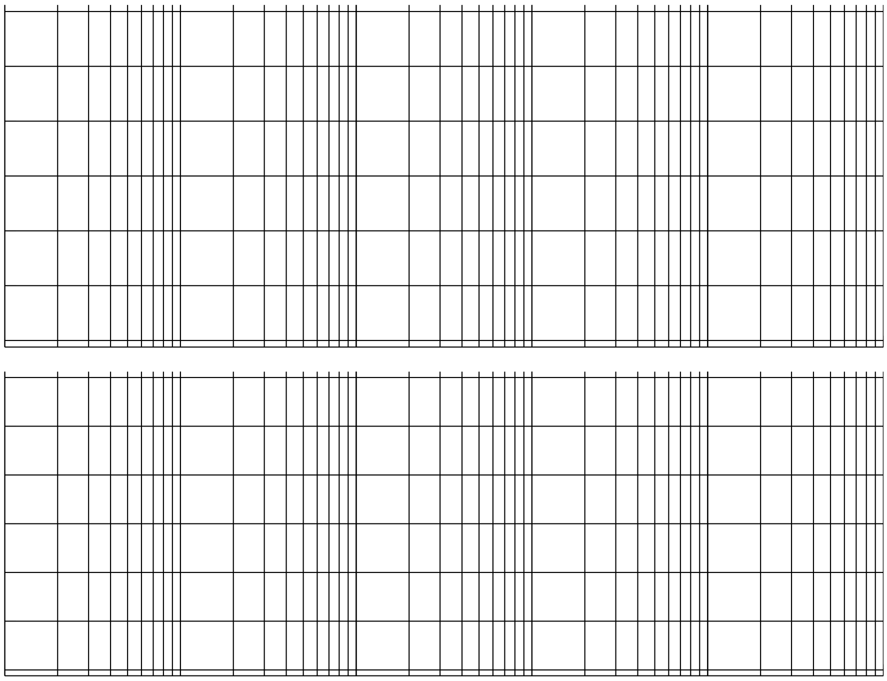
(d) $L(s) = \dfrac{100}{s\,(s+2)^2}$



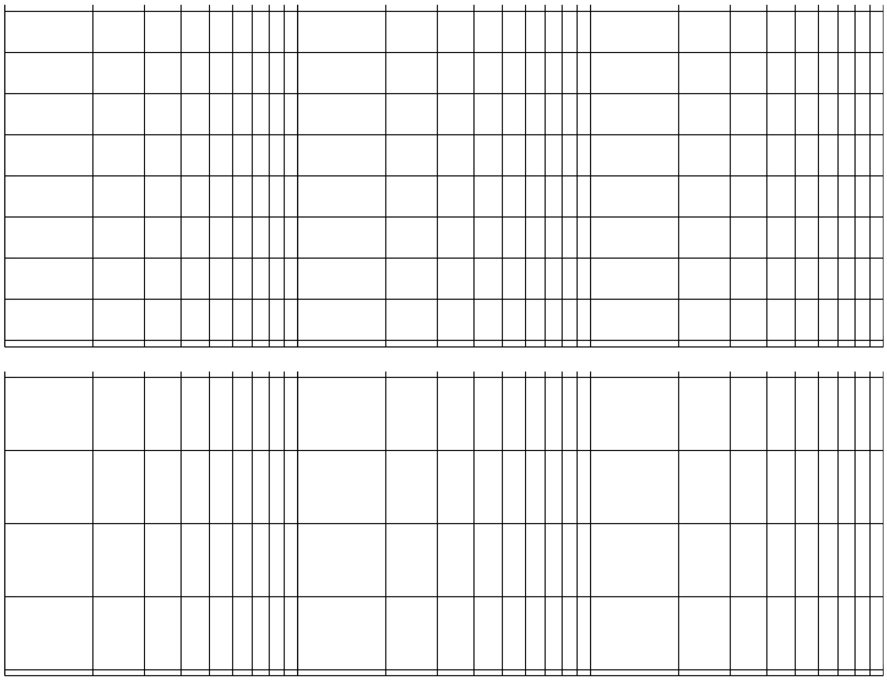
(e) $L(s) = \dfrac{100\,\mathrm{e}^{-\frac{s}{10}}}{s\,(s+10)^2}$



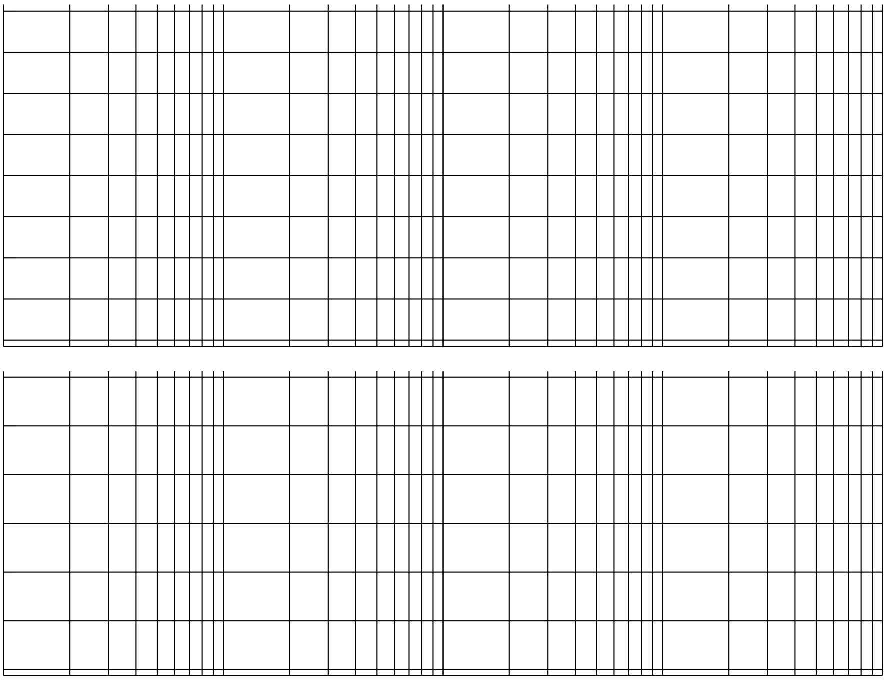
(f) $L(s) = \dfrac{40\ (s+10)}{s\,(s+2)^2\ (s+20)}$



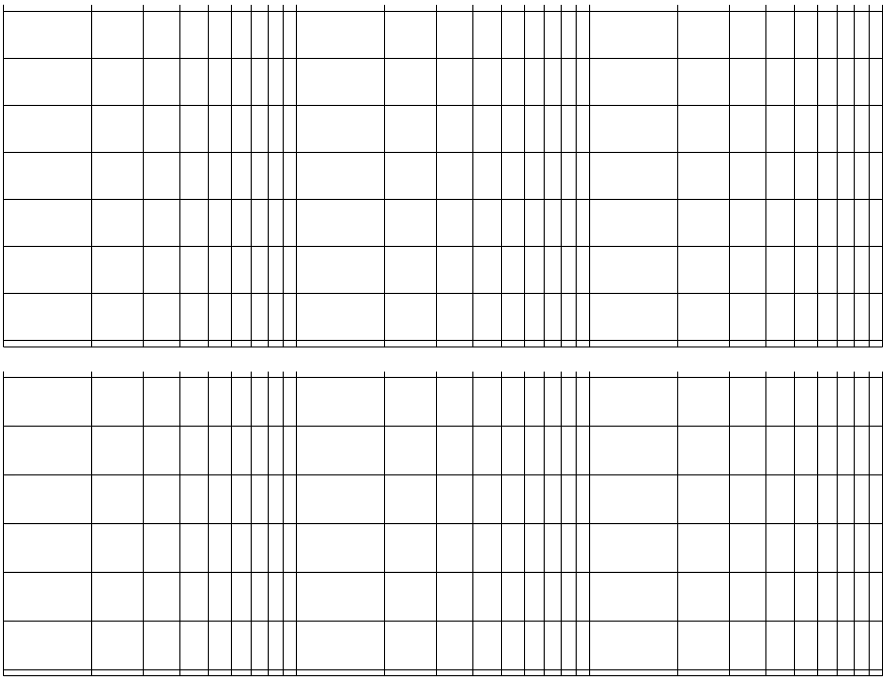
(g) $L(s) = \dfrac{10000\,s}{(s+10)^2\,(s+100)^2}$



(h) $L(s) = \dfrac{64\,s + 128}{s\,\left(s + \frac{1}{2}\right)\,\left(s^2 + \frac{13\,s}{4} + 64\right)}$



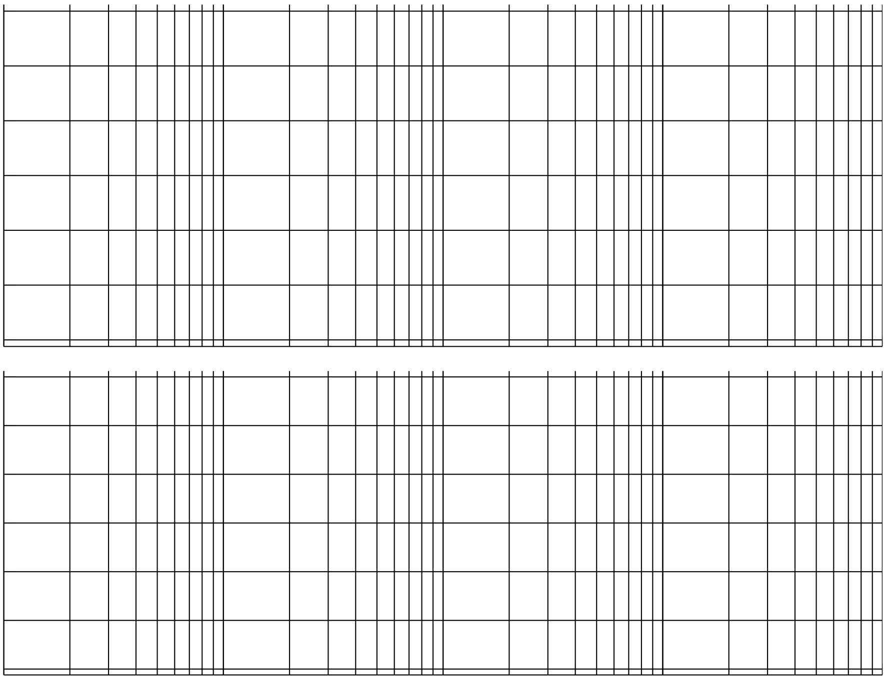
(i) $L(s) = \dfrac{s}{s^2\,(s+1)\,(s+5) + 10}$



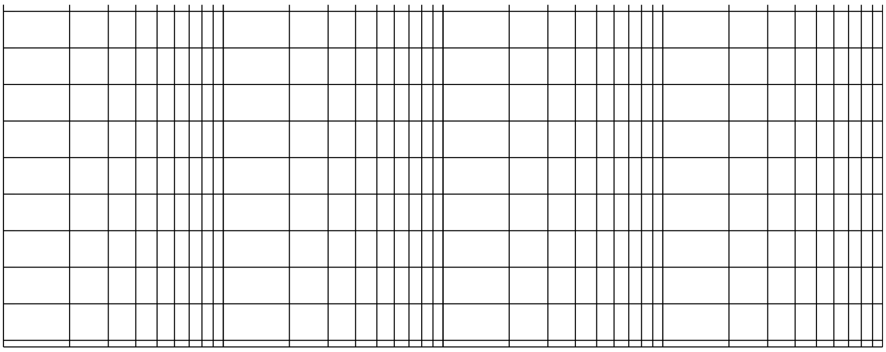
Aufgabe 6.4: [FPE15, 6.4]

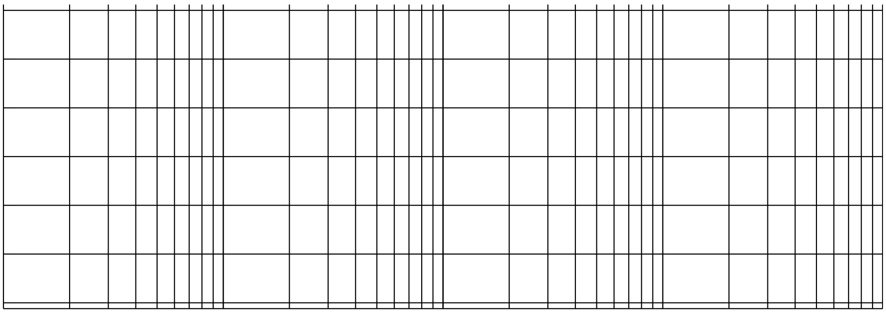
Skizzieren Sie die Amplituden- und Phasengänge (Asymptoten und ungefährer Verlauf) folgender Übertragungsfunktionen. Bestimmen Sie aus die Betrags- und die Phasenreserve sowie die Durchtrittskreis- und die Phasenschnittkreisfrequenz.

(a) $L(s) = \dfrac{1}{s\ (s+1)\ (s+10)\ (s+100)}$

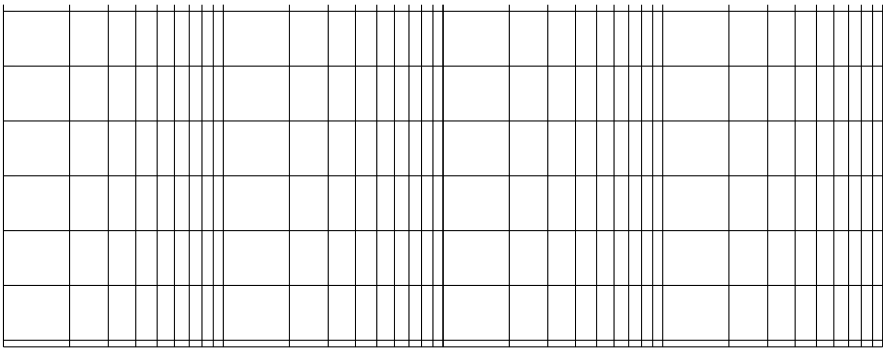


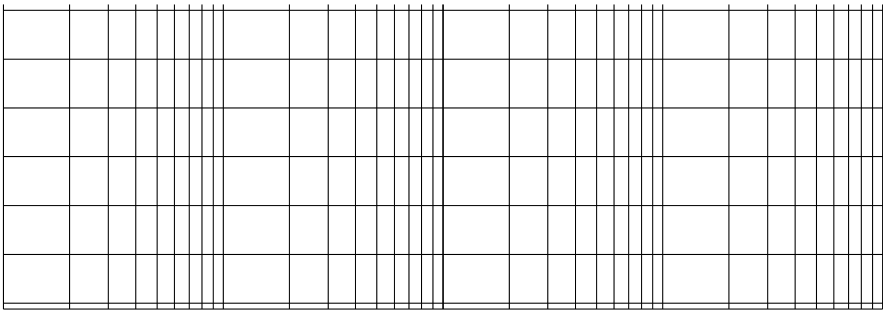
(b) $L(s) = \dfrac{s+5}{s\,(s+1)\,(s+10)\,(s+100)}$



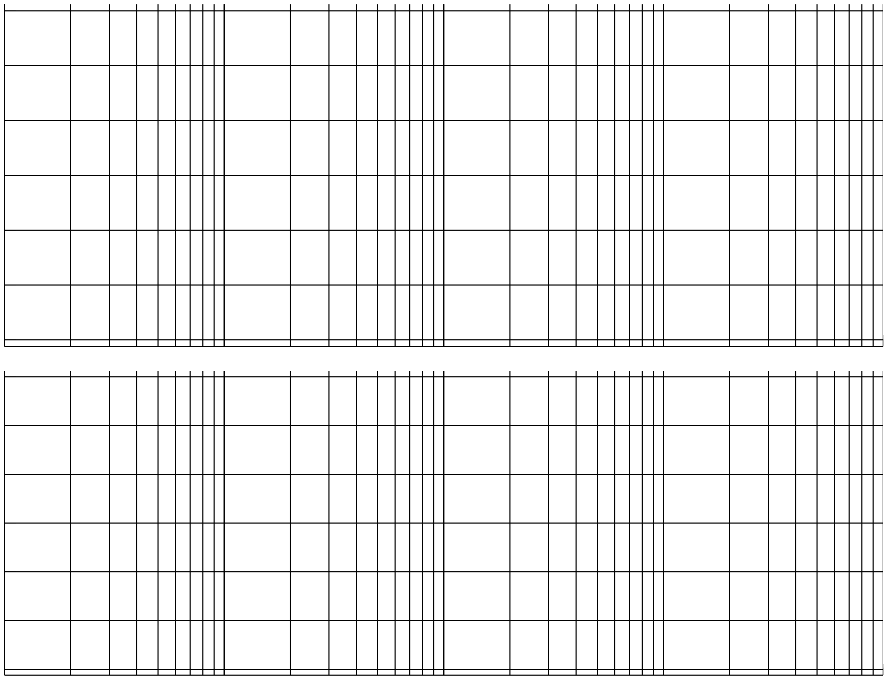


(c) $L(s) = \dfrac{(s+1)\,(s+3)}{s\,(s+1)\,(s+10)\,(s+100)}$





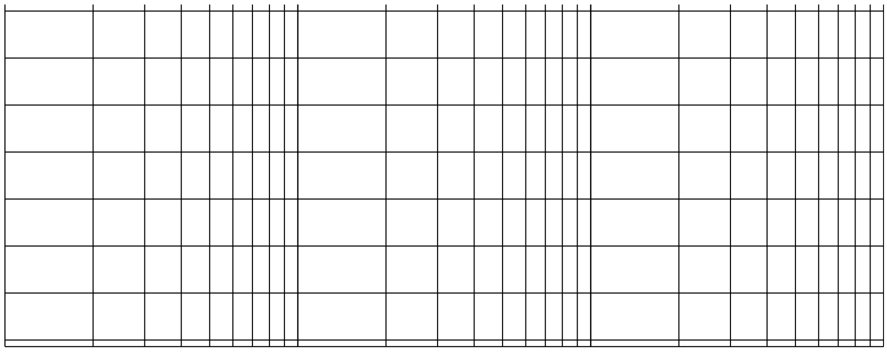
(d) $L(s) = \dfrac{(s+1)\ (s+3)\ (s+5)}{s\ (s+1)\ (s+10)\ (s+100)}$

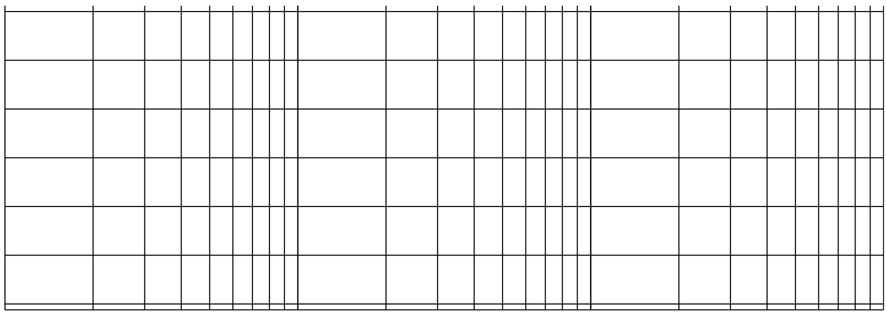


Aufgabe 6.5: [FPE15, 6.5] Komplexe Pole und Nullstellen

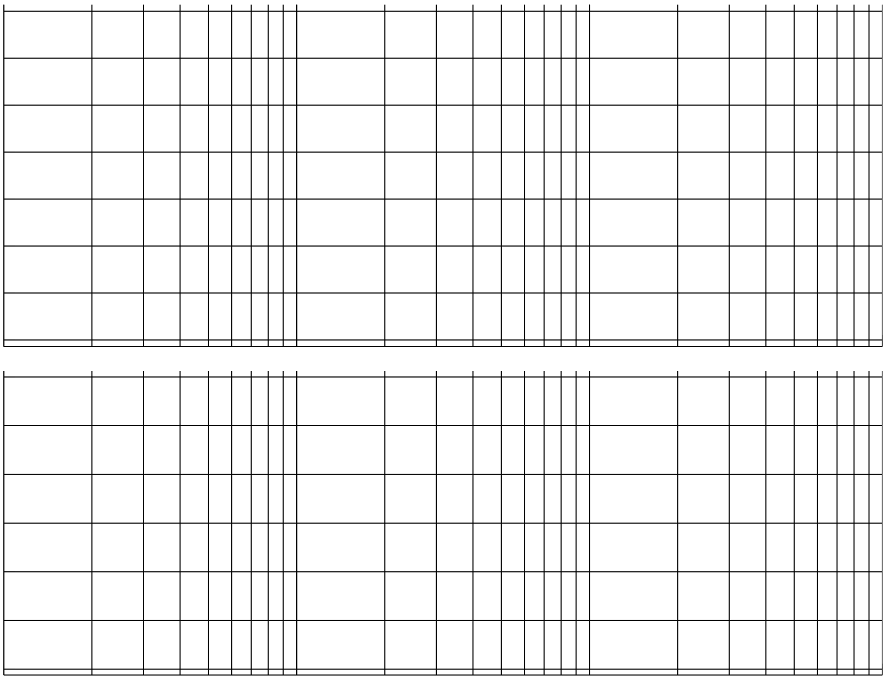
Skizzieren Sie die Amplituden- und Phasengänge (Asymptoten und ungefährer Verlauf) folgender Übertragungsfunktionen. Bestimmen Sie die Betrags- und die Phasenreserve.

(a) $L(s) = \dfrac{1}{s^2 + 2\,s + 5}$

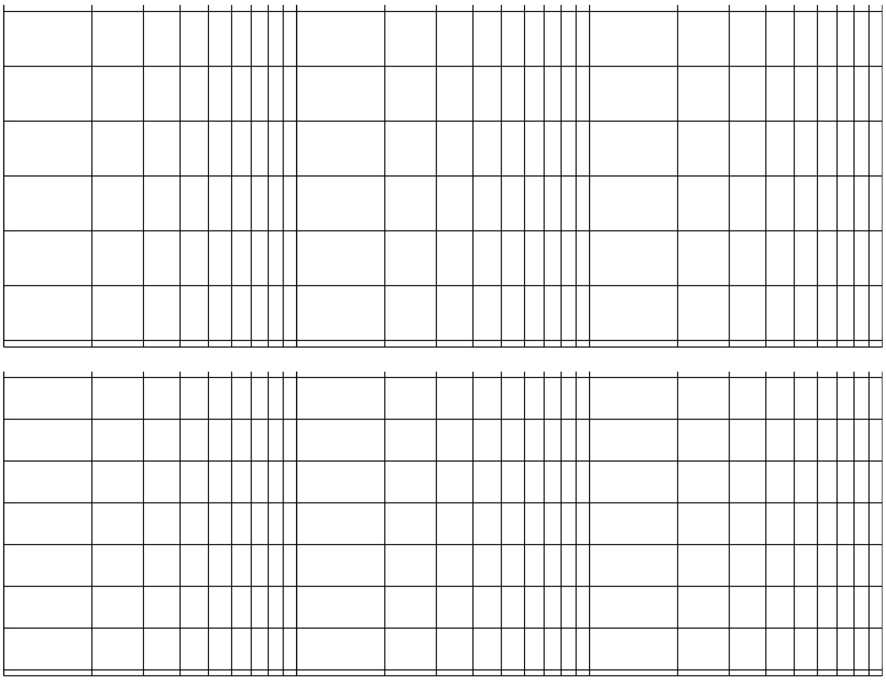




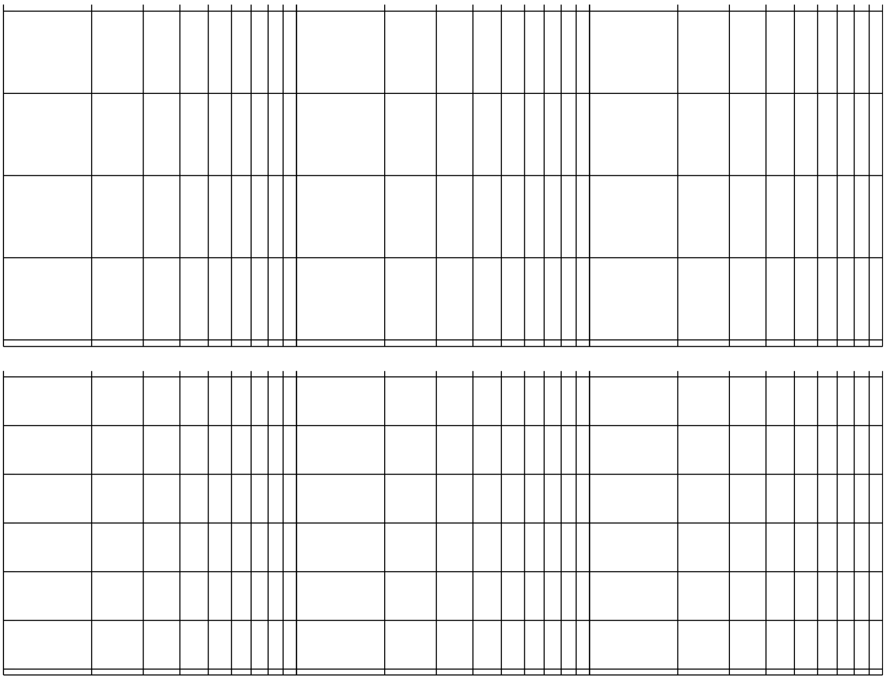
(b) $L(s) = \dfrac{1}{s\,\left(s^2 + 2\,s + 5\right)}$



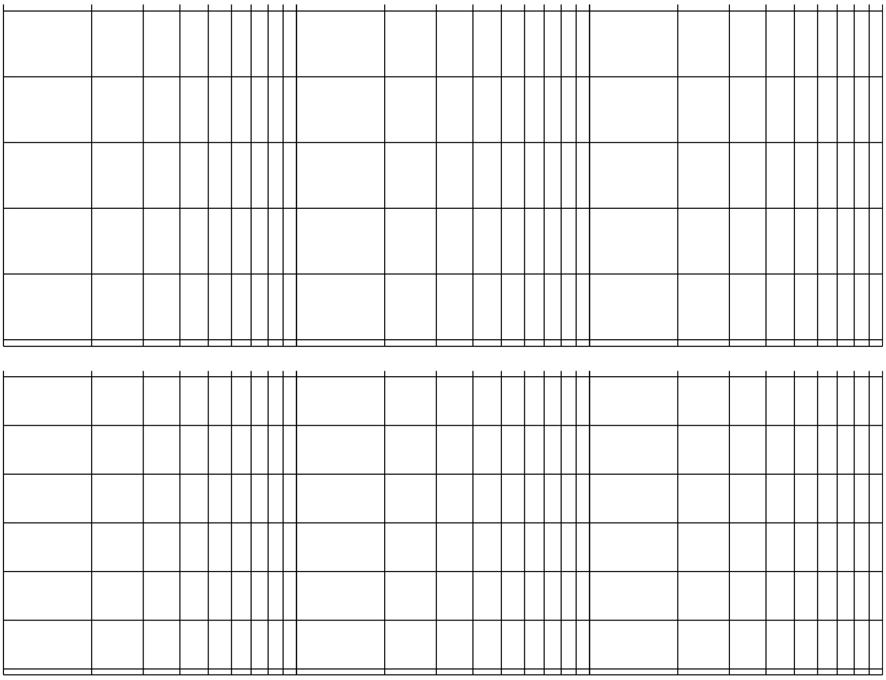
(c) $L(s) = \dfrac{s^2 + s + 3}{s\,\left(s^2 + 2\,s + 5\right)}$



(d) $L(s) = \dfrac{s}{s^2 + 2\,s + 5}$



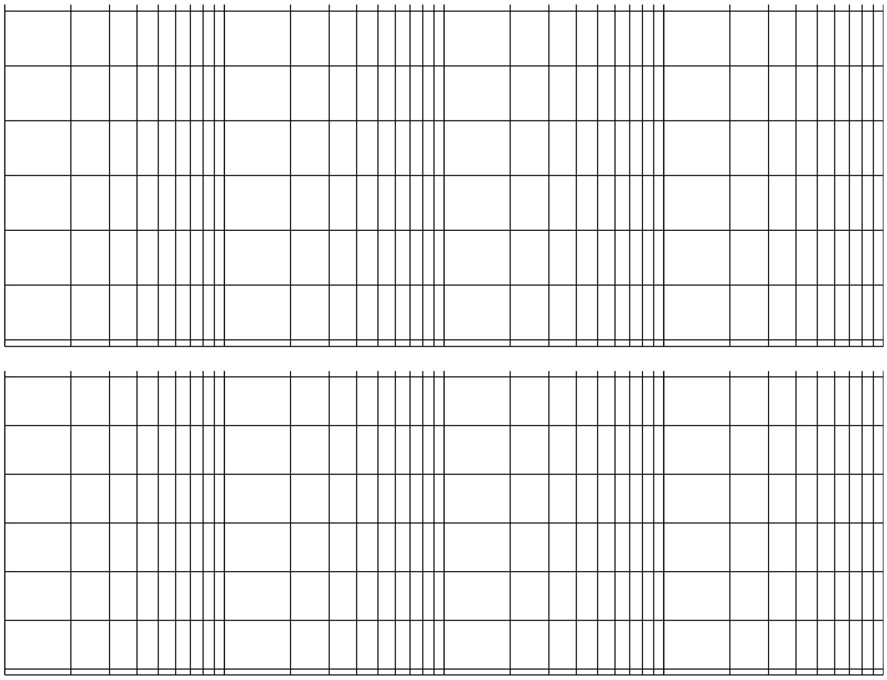
(e) $L(s) = \dfrac{s^2 + s + 1}{s^2 + 2\,s + 5}$



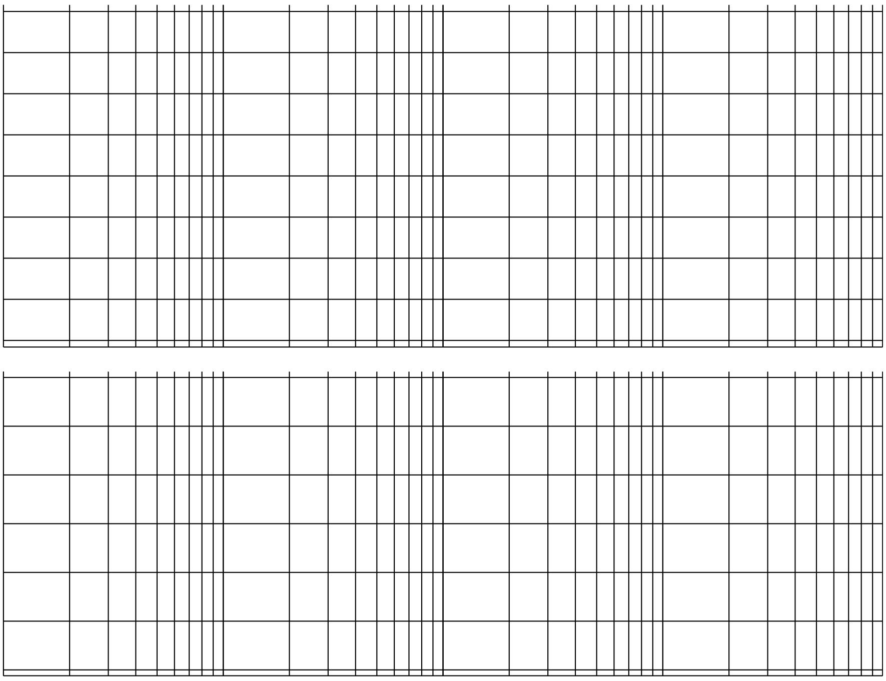
Aufgabe 6.6: [FPE15, 6.6] Mehrfache Pole im Ursprung

Skizzieren Sie die Amplituden- und Phasengänge (Asymptoten und ungefährer Verlauf) folgender Übertragungsfunktionen. Bestimmen Sie die Betrags- und die Phasenreserve. Berechnen Sie die Durchtrittskreis- sowie die Phasenschnittkreisfrequenz. Ist der geschlossene Einheitsregelkreis stabil?

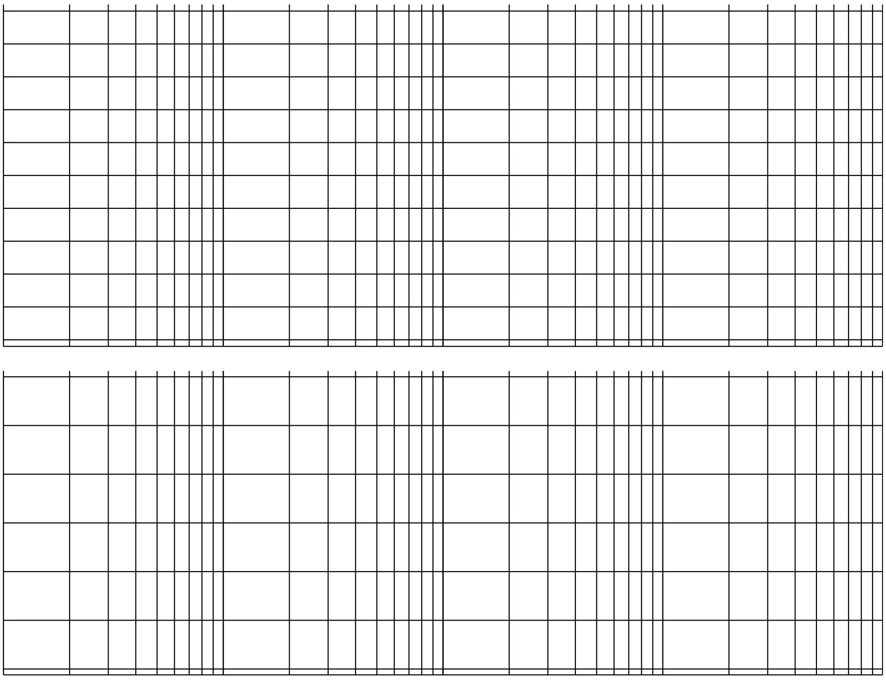
(a) $L(s) = \dfrac{1}{s^2\ (s+1)}$



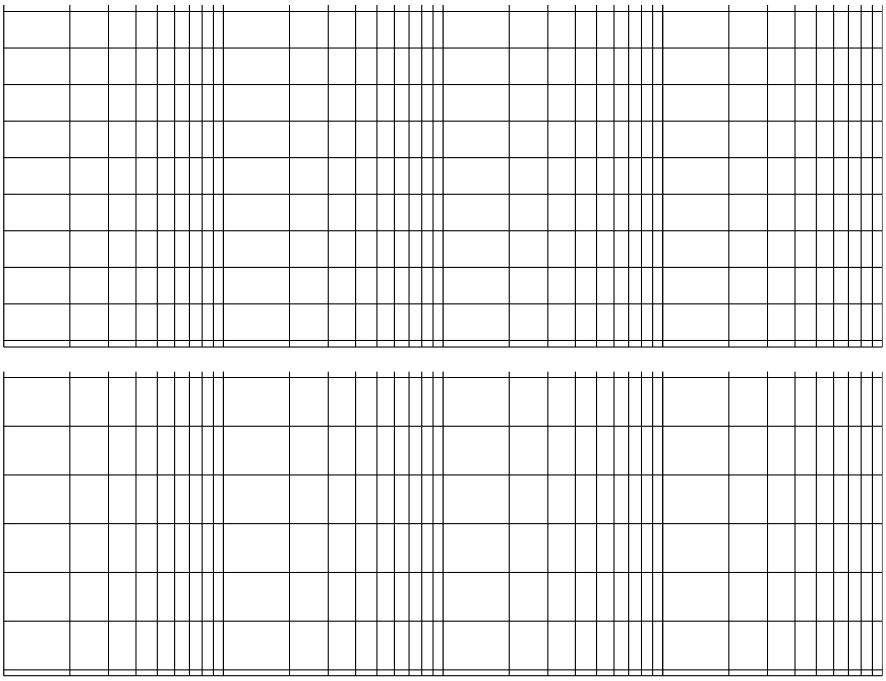
(b) $L(s) = \dfrac{1}{s^3\ (s+1)}$



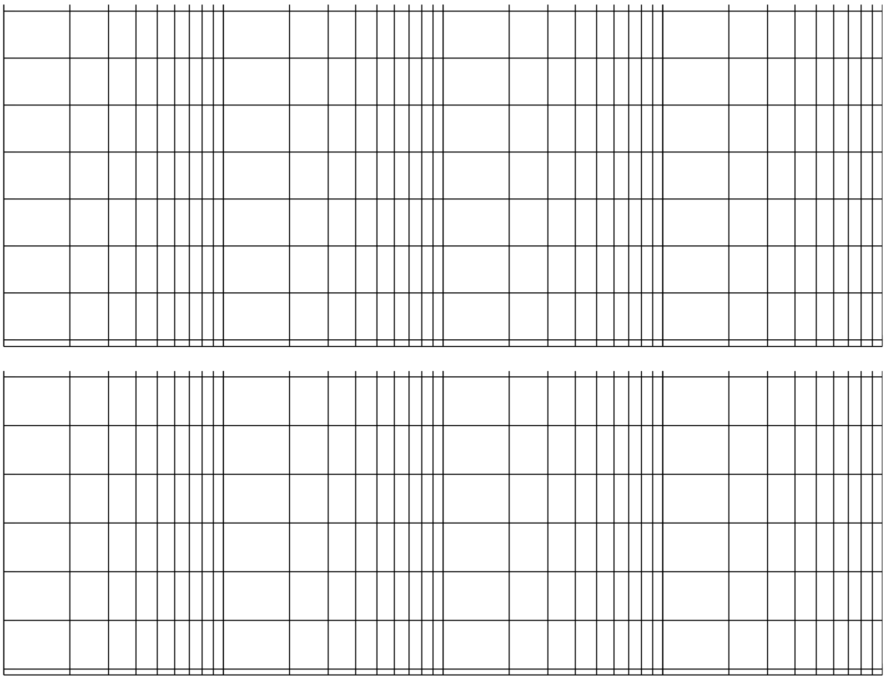
(c) $L(s) = \dfrac{1}{s^4\ (s+1)}$



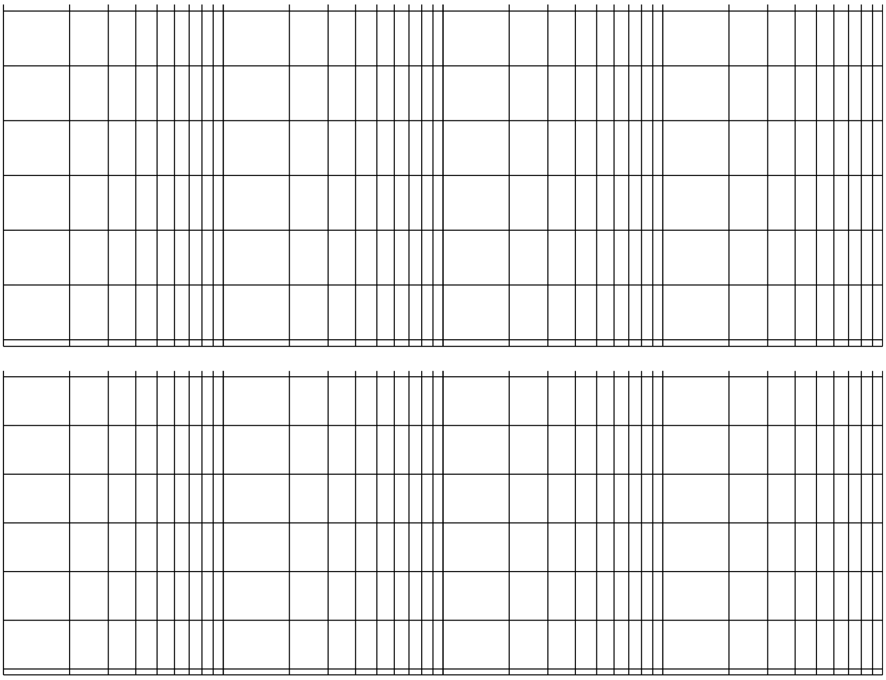
(d) $L(s) = \dfrac{s+3}{s^2\ (s+10)}$



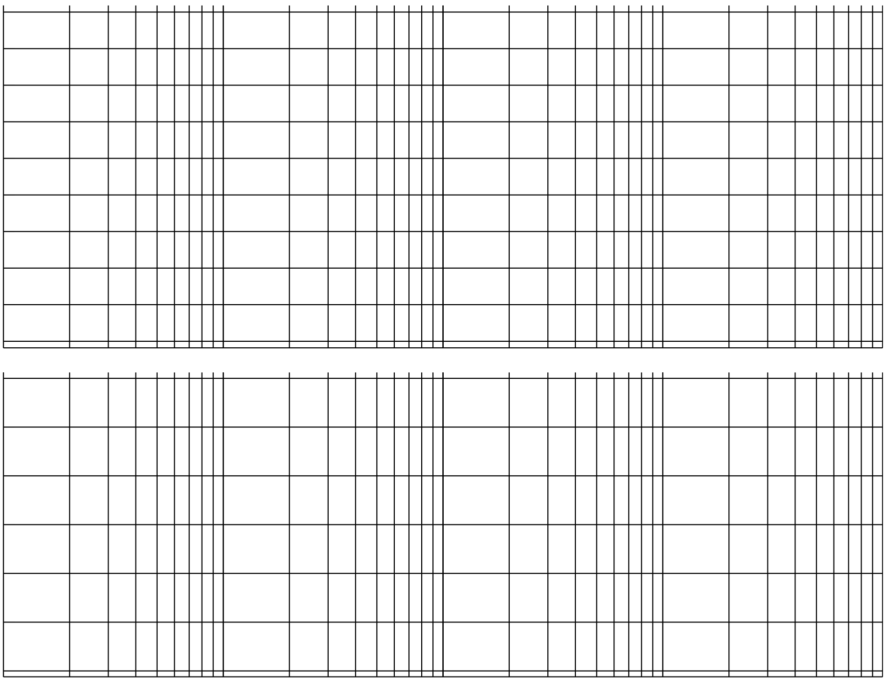
(e) $L(s) = \dfrac{(s+1)^2}{s^2\ (s+3)}$



(f) $L(s) = \dfrac{(s+1)^2}{s^3\,(s+3)}$



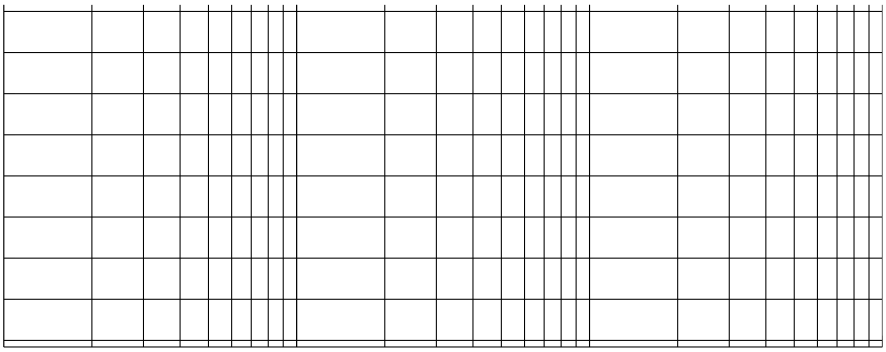
(g) $L(s) = \dfrac{(s+1)^2}{s^2\,(s+3)^2}$

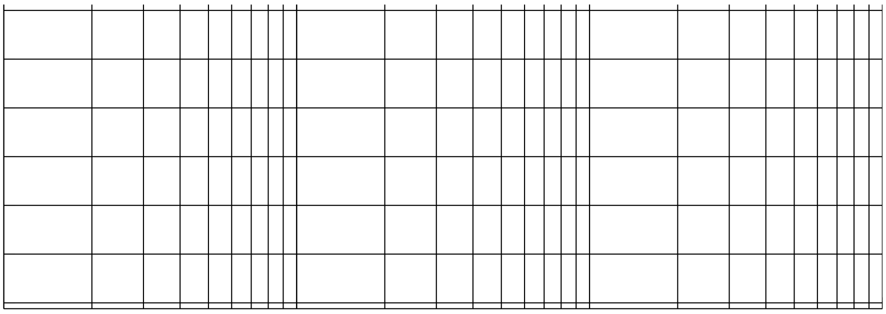


Aufgabe 6.7: [FPE15, 6.7] Reelle und kommplexe Pole gemischt

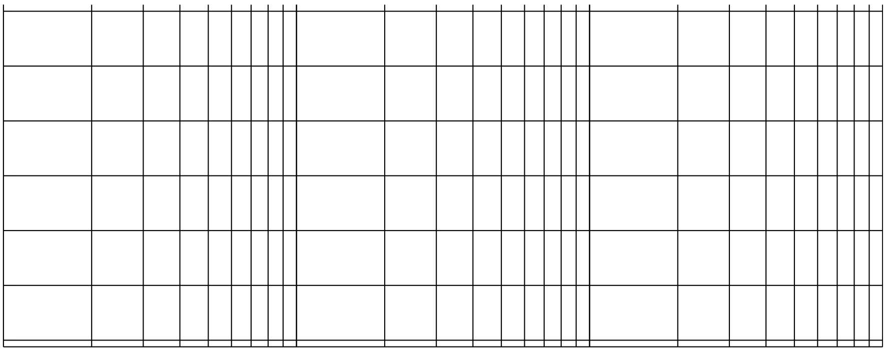
Skizzieren Sie die Amplituden- und Phasengänge (Asymptoten und ungefährer Verlauf) folgender Übertragungsfunktionen. Bestimmen Sie die Betrags- und die Phasenreserve. Berechnen Sie die Durchtrittskreis- sowie die Phasenschnittkreisfrequenz. Ist der geschlossene Einheitsregelkreis stabil?

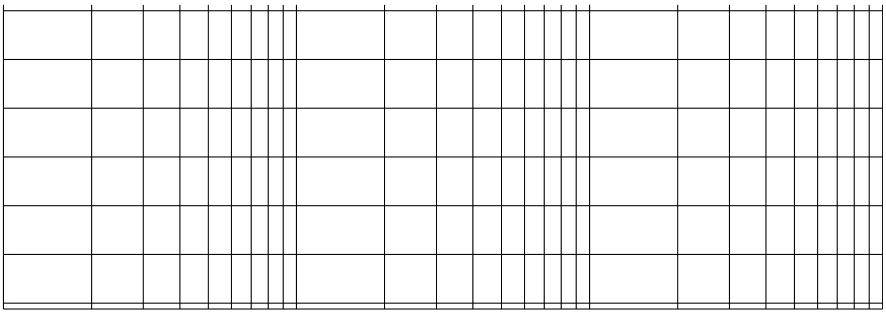
(a) $L(s) = \dfrac{s+1}{s\,(s+2)\,\left(s^2+s+1\right)}$



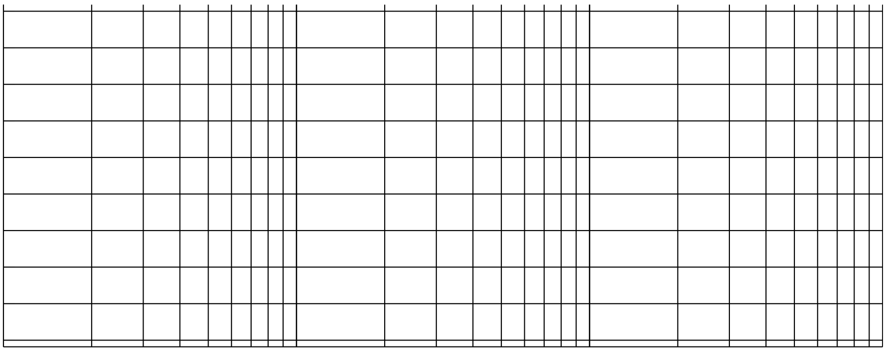


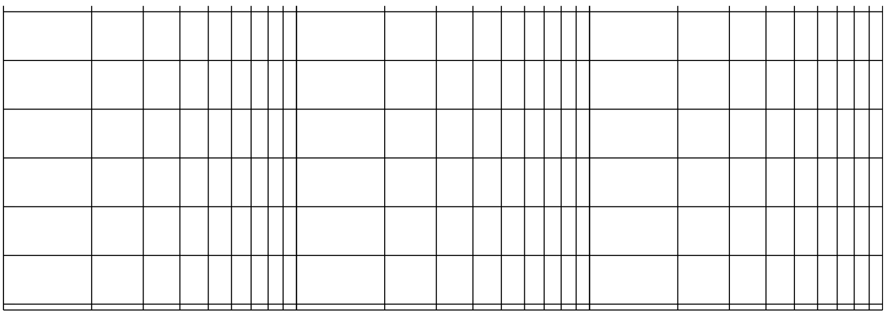
(b) $L(s) = \dfrac{s+1}{s^2\ (s+2)\ \left(s^2+s+1\right)}$



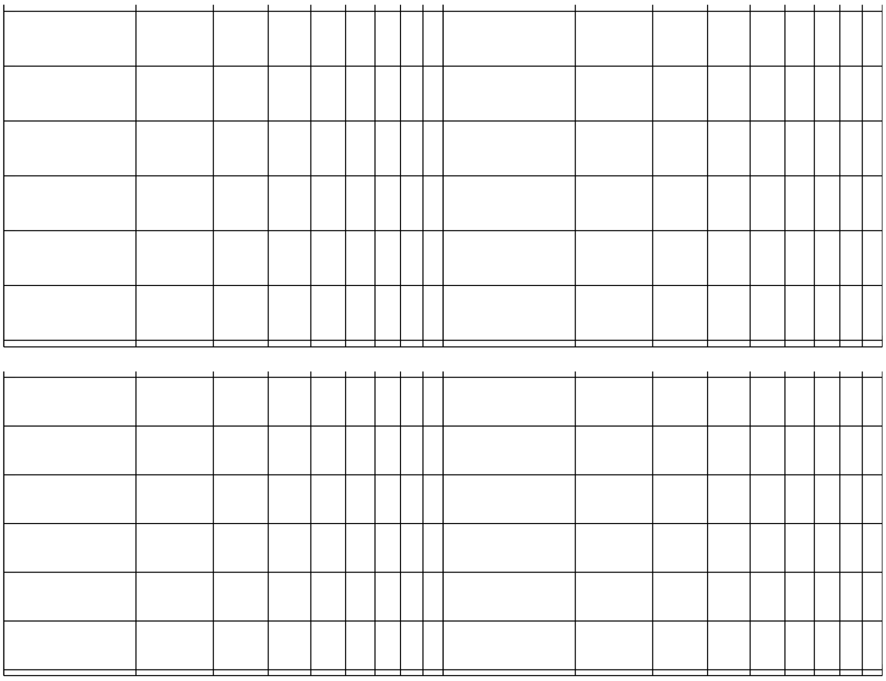


(c) $L(s) = \dfrac{(s+1)^2}{s^2\ (s+2)\ \left(s^2+s+1\right)}$

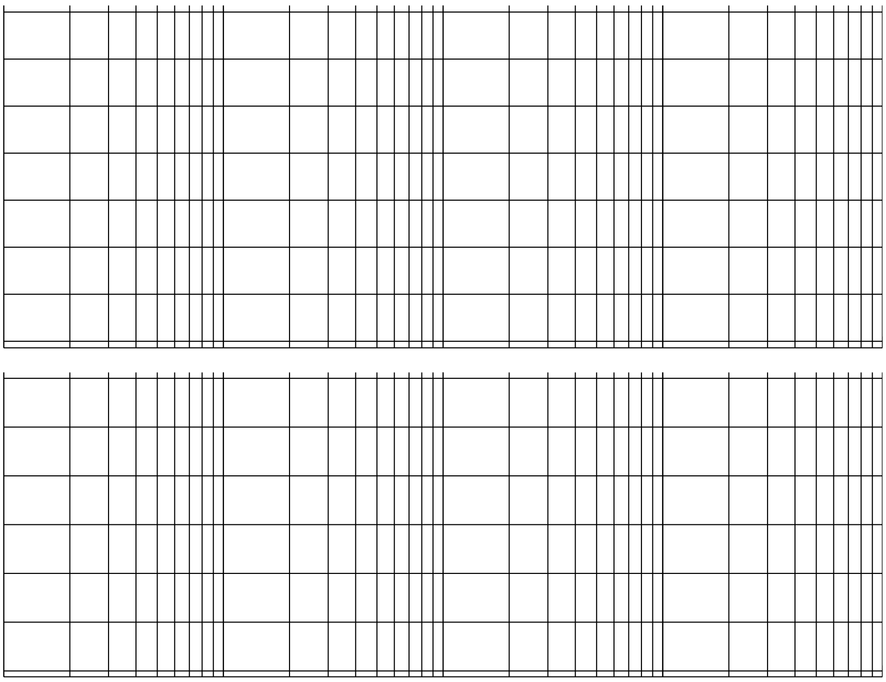




(d) $L(s) = \dfrac{(s+1)\,\left(s^2+2\,s+4\right)}{s^2\,(s+2)\,\left(s^2+s+1\right)}$



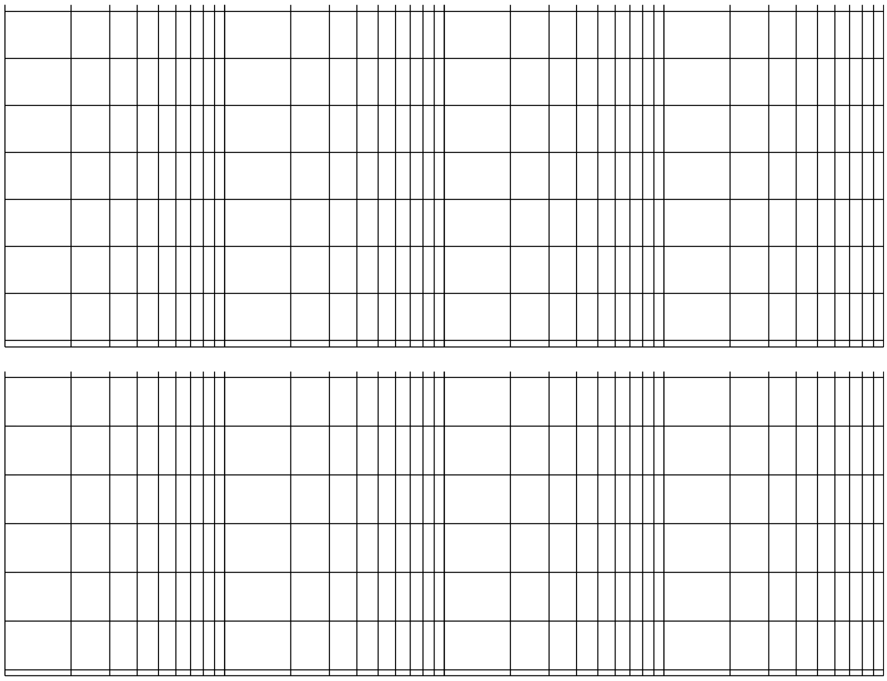
(e) $L(s) = \dfrac{s^2+2\,s+4}{s^2\,(s+2)\,\left(s^2+s+1\right)}$



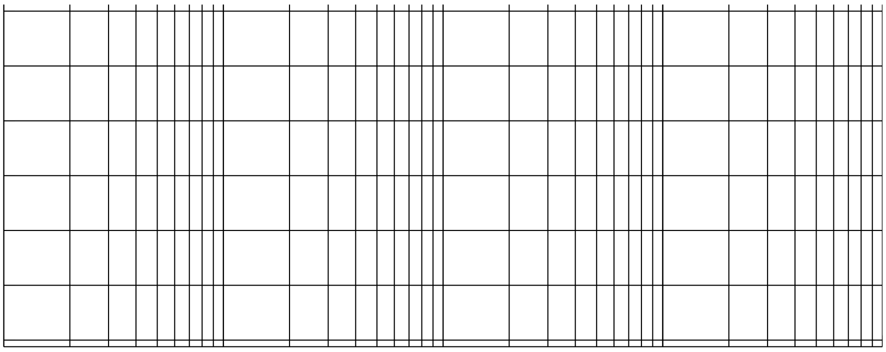
Aufgabe 6.8: [FPE15, 6.8] RHE Pole und Nullstellen

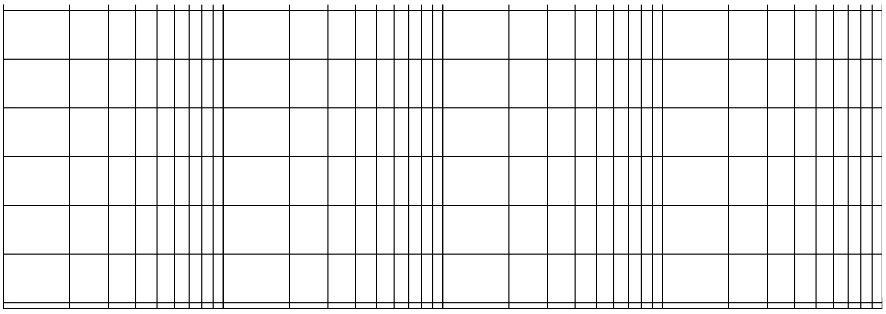
Skizzieren Sie die Amplituden- und Phasengänge (Asymptoten und ungefährer Verlauf) folgender Übertragungsfunktionen. Bestimmen Sie die Betrags- und die Phasenreserve. Berechnen Sie die Durchtrittskreis- sowie die Phasenschnittkreisfrequenz. Ist der geschlossene Einheitsregelkreis stabil?

(a) $L(s) = \dfrac{s+1}{\left(s^2-9\right)\,(s+5)}$

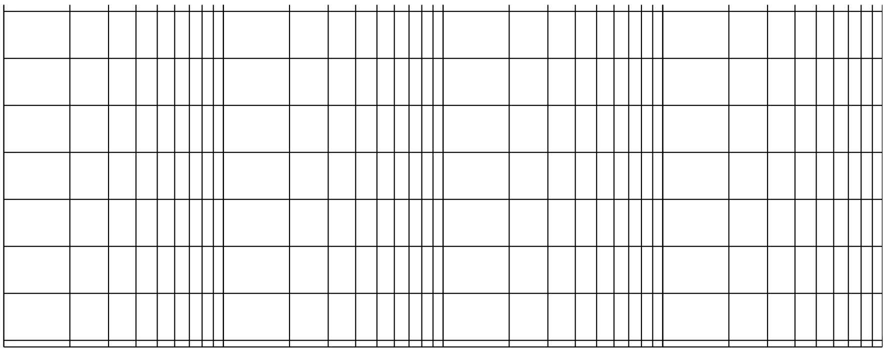


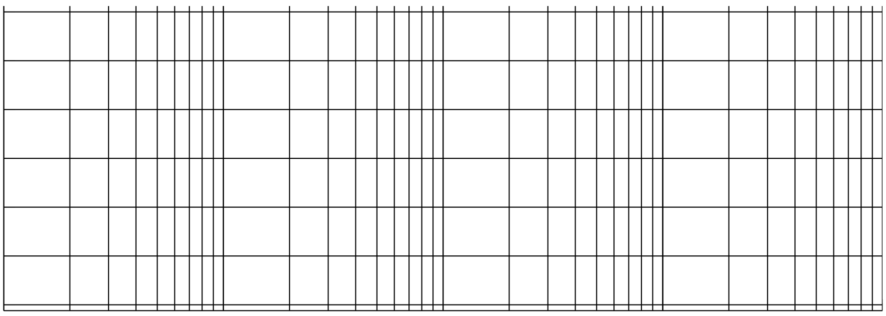
(b) $L(s) = \frac{s+1}{s^3}$



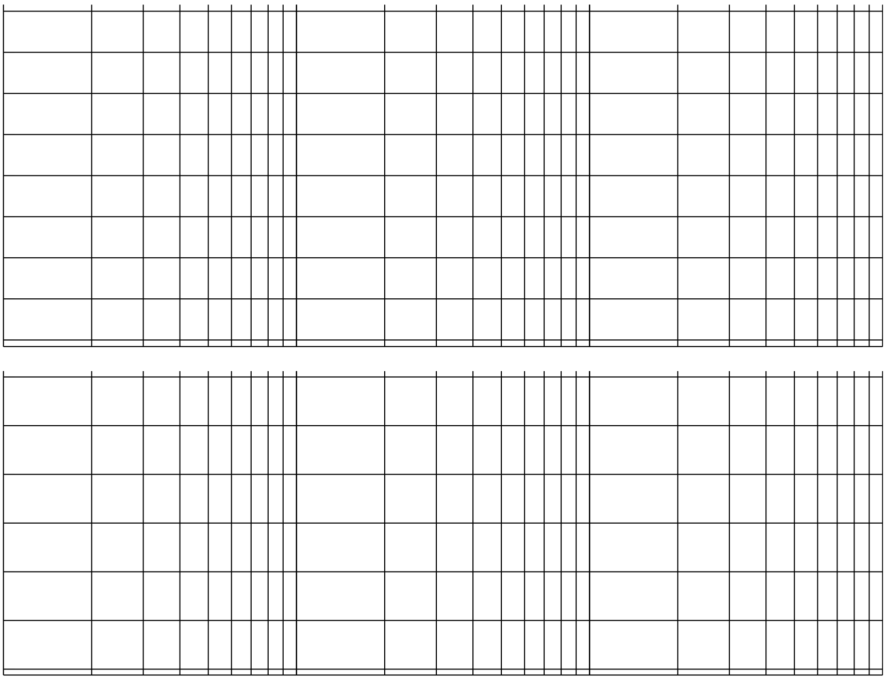


(c) $L(s) = \frac{s^2 + s + 3}{s^2\ (s+2)\ \left(s^2 + s + 1\right)}$

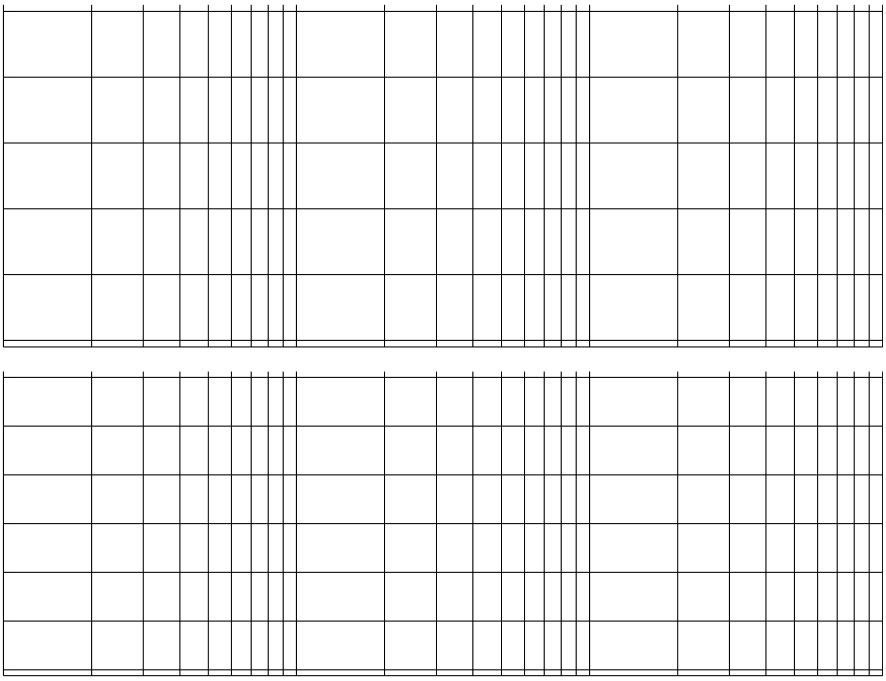


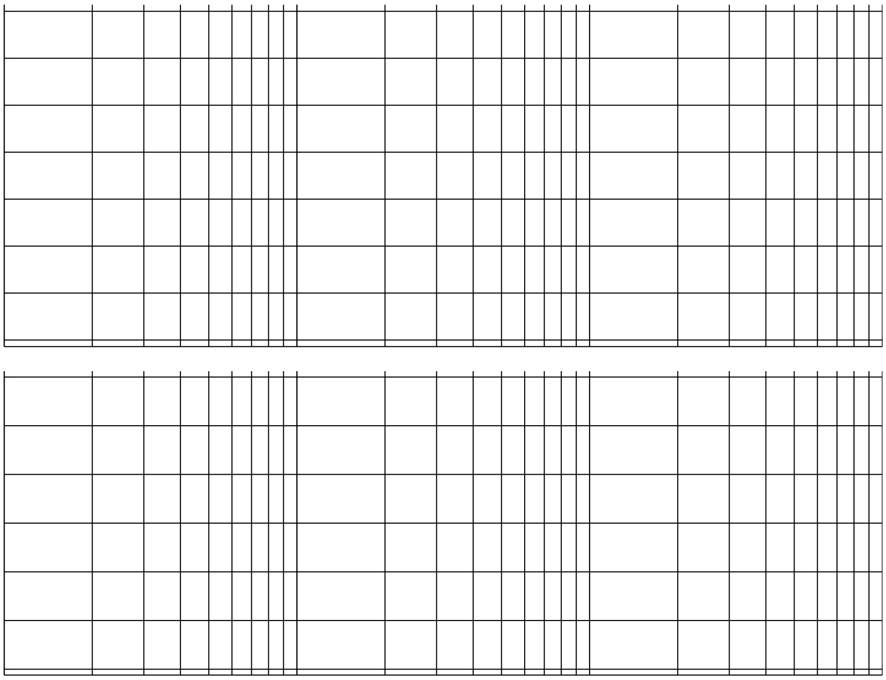


(d) $L(s) = \dfrac{s+1}{s\ (s+2)\ \left(s^2+s+1\right)}$



(e) $L(s) = \dfrac{s+3}{s\ (s+1)\ (s+2)}$

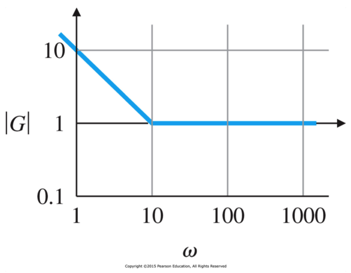




(f) $L(s) = \dfrac{1}{(s+2)\,(s-5)}$

Aufgabe 6.9: [FPE15, 6.9]

Bestimmen sie die Übertragungsfunktion für folgendes asymptotisches Bodediagramm:



[FPE15, Figure 6.85]

Bestimmen Sie die Sprungantwort des Systems.